

GE Theory – Assignment 2

Question 1 (16 points) Subsistence in the Edgeworth Box

There are two consumers with the following utility functions:

$$u_1(x_{11}, x_{21}) = \begin{cases} (x_{11} - 1)^{\frac{3}{4}} (x_{21} - 3)^{\frac{1}{4}} & \text{if } x_{11} \geq 1 \text{ and } x_{21} \geq 3 \\ -1 & \text{otherwise} \end{cases}$$

$$u_2(x_{12}, x_{22}) = \begin{cases} (x_{12} - 3)^{\frac{1}{4}} (x_{22} - 1)^{\frac{3}{4}} & \text{if } x_{12} \geq 3 \text{ and } x_{22} \geq 1 \\ -1 & \text{otherwise} \end{cases}$$

The initial endowment for consumer 1 is $\omega_1 = (3, 3)$ and for consumer 2 it is $\omega_2 = (3, 5)$. There is a single firm with the production set $Y_1 = \mathbb{R}_-^2$ (free disposal) and the ownership shares are $\theta_{11} = \theta_{21} = \frac{1}{2}$.

- (i) [4 points:] Calculate the equilibrium price and allocation in this economy, normalizing $p_1 = 1$.
- (ii) [4 points:] Find the Pareto set and contract curve. Define them formally.
- (iii) [4 points:] Sketch the Edgeworth box for this economy. Include each of the following in your sketch: the endowment, the equilibrium allocation, the budget line at the equilibrium price, the Pareto set, the contract curve, and the indifference curves for each consumer at the utility from consuming their endowment.
- (iv) [4 points:] Suppose instead of a market for the two goods, Consumer 2 can make Consumer 1 a take-it-or-leave-it offer. What is the optimal offer for Consumer 2 to give to Consumer 1? Assume Consumer 1 cannot make a counter-offer to Consumer 2's offer and will accept an offer if they are indifferent to accepting or rejecting.

Question 2 (20 points) Bliss Points

In a pure-exchange economy with $I > 1$ consumers and $X_i = \mathbb{R}_+^L$ for all i , each \succeq_i is complete, transitive, continuous, and strictly convex. There is a single firm with only the free disposal technology, $Y_1 = \mathbb{R}_-^L$. Consumers have an initial endowment $\omega_i \in \mathbb{R}_+^L$ and $\theta_{i1} = \frac{1}{I}$ for all i .

Note that we do *not* assume local nonsatiation at every $x_i \in X_i$. We define a *bliss point* for consumer i as a bundle $x_i^B \in X_i$ satisfying $x_i^B \succeq_i x_i$ for all $x_i \in X_i$.

- (i) [4 points] Show that for any consumer satisfying the assumptions on preferences above, there can be at most one bliss point.

Based on part (i), you can conclude that preferences are locally nonsatiated at all points $x_i \in X_i$, where $x_i \neq x_i^B$. That is, preferences are locally nonsatiated at any point that isn't the consumer's unique bliss point.

For parts (ii)-(iv), (x^*, y^*, p^*) is a competitive equilibrium of this economy.

- (ii) [4 points] Show that for all $x_i \in X_i$ satisfying $x_i \succ_i x_i^*$ we have that $p^* \cdot x_i > p^* \cdot \omega_i + \theta_{i1} p^* \cdot y_1^*$.
- (iii) [4 points] Show that for all $x_i \in X_i$ satisfying $x_i \succeq_i x_i^*$ we have that $p^* \cdot x_i \geq p^* \cdot \omega_i + \theta_{i1} p^* \cdot y_1^*$. *Hint:* You should consider the cases where x_i^* is and is not a bliss point separately.
- (iv) [4 points] Using (ii) and (iii), show that (x^*, y^*) is Pareto optimal.
- (v) [4 points] In class, our proof for the existence of a Walrasian equilibrium assumed locally nonsatiated preferences for all consumers, which we do not assume here. Come up with an example private-ownership economy satisfying all of the assumptions above that has no equilibrium. You may use an Edgeworth box diagram as part of your answer.

Question 3 (14 points) Equilibrium Nonexistence

There are 2 consumers and 2 goods in a pure exchange economy. Consumer 1 and 2's preferences are as follows (where $x_{\ell i}$ is i 's consumption of the ℓ th good):

1: $x'_1 \succ_1 x_1$ iff either (i) $x'_{11} > x_{11}$ or (ii) $x'_{11} = x_{11}$ and $x'_{21} > x_{21}$ and $x'_1 \sim_1 x_1$ iff $x'_1 = x_1$.

2: $x'_2 \succ_2 x_2$ iff $x'_{12} > x_{12}$ and $x'_2 \sim_2 x_2$ iff $x'_{12} = x_{12}$.

There is a single firm with $Y_1 = -\mathbb{R}_+^2$. Assume $\omega_{\ell i} > 0 \forall \ell, i$ and $\theta_{11} = \theta_{21} = \frac{1}{2}$.

- (i) [4 points:] What is the Pareto set?
- (ii) [5 points:] Show that a Walrasian equilibrium does not exist for this economy.
- (iii) [5 points:] In class, we saw that if $z(p)$ was defined over all nonzero, nonnegative price vectors $p \in \mathbb{R}_+^L$ and satisfied continuity, homogeneity of degree zero and Walras' law, then there is a price vector p^* such that $z(p^*) \leq 0$. Which condition is not satisfied in this example? Give a counterexample to show that this condition does not hold.