GE Theory – Assignment 1

Question 1 (20 points) Partial Equilibrium

In a two-good economy there are 4 consumers and 4 producers. Consumer i, $i = 1, \ldots, 4$, owns $\omega_{mi} = 10i$ units of the numeraire and has utility $u_i(x_i, m_i) = 2i\sqrt{x_i} + m_i$ if consuming m_i units of the numeraire good and $x_i \geq 0$ units of the commodity under consideration. Firm j, $j = 1, \ldots, 4$, produces the commodity q_j from the inputs of the numeraire z_j according to the following technology:

$$Y_j = \left\{ \left(-z_j, q_j\right) : q_j \geq 0 \text{ and } z_j \geq j q_j \right\}$$

Every consumer has an equal share in each firm, i.e. $\theta_{ij} = \frac{1}{4}, \forall i, j$.

(i) **[4 points]** Determine the equilibrium price and allocation, i.e. each (m_i, x_i) for each consumer and q_j for each firm.

(ii) **[8 points]** Now assume there is a social planner that has the following problem:

$$\max_{(x_1,\ldots,x_4,m_1,\ldots,m_4,z_1,\ldots,z_4,q_1,\ldots,q_4)} \sum_{i=1}^{4} \alpha_i \log\left(u_i\left(x_i, m_i\right)\right)$$

subject to the nonnegativity, technology and resource constraints in the economy, where α_i is the weight that the social planner places on consumer i (with $\sum_{i=1}^{4} \alpha_i = 1$) and $\log(\cdot)$ is the natural logarithm. Formally state the full social planner's problem and solve for the optimal allocation of (x_i, m_i) for each i as a function of α_i.

(iii) **[4 points]** Provide the set of weights α_i that would yield the competitive equilibrium in (i) as a social welfare optimum.

(iv) **[4 points]** Suppose now that $\alpha_i = \frac{1}{4}$ for all i in the social planner problem in (ii). Determine the transfers of the endowments needed to achieve this goal in a competitive equilibrium.

Question 2 (12 points) Bilateral Negative Externality

Two consumers $i = 1, 2$ live together. Consumer 1 likes to listen to loud bagpipe music on a record player that consumer 2 does not like. Let $h \geq 0$ denote the volume setting on consumer 1's record player and m_i consumer i's consumption of the composite commodity.

The two utility functions are:

$$u_1(h, m_1) = 10h - h^2 + m_1$$
$$u_2(h, m_2) = -4h + m_2$$

Each consumer i is endowed with $\omega_{mi} = 20$ units of the composite commodity.

(i) **[2 points]** Determine the quantities $\left(h^\star, m_1^\star, m_2^\star\right)$ and the utilities $u_1^\star, u_2^\star$ in the unregulated competitive outcome.

(ii) **[2 points]** If the social planner seeks to maximize the sum of the two consumers' utilities, determine the quantity h° and the social welfare $u_1^\circ + u_2^\circ$.

(iii) **[4 points]** Suppose now there is a per-unit tax on h. How should the tax be set to arrive at the Pareto optimal level of h as in (ii)?

(iv) **[4 points]** Suppose now consumer 2 has the right to a quiet apartment ($h = 0$). If consumer 2 was to offer consumer 1 a take-it-or-leave-it deal to play music at volume $\widetilde{h}$ given a transfer $\widetilde{T}$, what level of $\widetilde{h}$ and $\widetilde{T}$ should consumer 2 offer to maximize their utility?

Question 3 (18 points) Tradeable Permits

There are J firms each with the derived profit function:

$$\pi_j\left(h_j\right) = jh_j - \frac{1}{2}h_j^2$$

There are I consumers each with the identical derived utility function:

$$\phi_i\left(h\right) = -h$$

where $h = \sum_{j=1}^{J} h_j$.

(i) **[5 points]** What is the competitive outcome ($h_j^\star$ for each firm and $h^\star$)?

(ii) **[5 points]** What is the Pareto optimal outcome (h_j° for each firm and h°)?

(iii) **[4 points]** Suppose there were tradable permits given to each firm to produce $\widetilde{h}_j = \frac{h^\circ}{J}$ units of the externality, where h° is equal to the total externality in part (ii). What is the aggregate demand for permits as a function of the price?

(iv) **[4 points]** What is the equilibrium price of a permit?