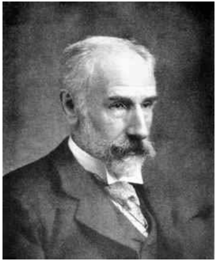
The Edgeworth Box

230333 Microeconomics 3 (CentER) – Part II
Tilburg University

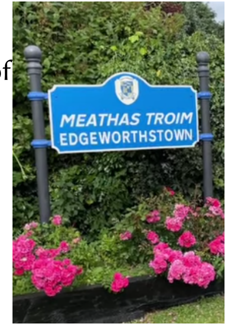
Introduction

- We will now study how prices are determined in general equilibrium in an economy with 2 goods and 2 consumers.
- We will first study this graphically using a useful tool called the *Edgeworth Box*.

Francis Ysidro Edgeworth (1845-1926)



- Born in Edgeworthstown, County Longford, Ireland and educated at Trinity College Dublin.
- Famous work is *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences* (1881).
- Invented the contract curve before the notion of Pareto optimality. His work did not actually have a diagram of a “box” but when Pareto developed it he attributed it to him. Later, Sir Arthur Lyon Bowley’s text popularized the Edgeworth box and it is therefore sometimes known as the Edgeworth-Bowley Box.
- Drew the first indifference curves and with a convex shape hinted towards the notion of diminishing marginal utility.
- Was the first editor of the *Economic Journal*.

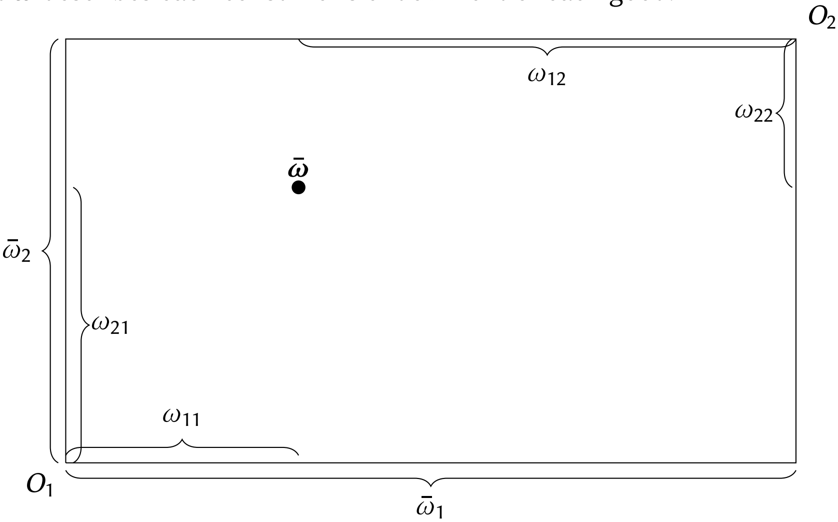


Edgeworth Box: $I = L = 2$ with Pure Exchange

- Two consumers $i = 1, 2$ and two commodities $\ell = 1, 2$.
- Consumer i has preferences $\succeq_i$ over bundles $\boldsymbol{x}_i = \left(x_{1i,} x_{2i}\right)$.
- Consumer i has an endowment $\boldsymbol{\omega}_i = (\omega_{1i}, \omega_{2i})$.
- The total endowment in the economy is $\bar{\boldsymbol{\omega}} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$.
- There is one firm with a production set $Y_1 = \mathbb{R}^2_-$ (*free disposal*).
- An *allocation* $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2) \in \mathbb{R}^4_+$ is *feasible* if $x_{\ell 1} + x_{\ell 2} \leq \bar{\omega}_\ell, \forall \ell = 1, 2$.
- If $x_{\ell 1} + x_{\ell 2} = \bar{\omega}_\ell, \forall \ell$, an allocation is *nonwasteful* (no disposal).
- All nonwasteful allocations can be represented in an *Edgeworth box*.

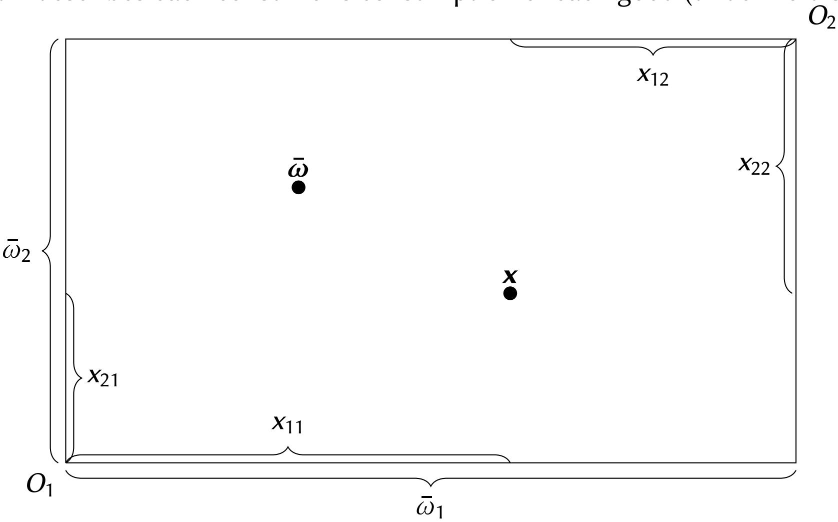
Edgeworth Box: Endowments

The point $\bar{\boldsymbol{\omega}}$ describes each consumer's endowment of each good.



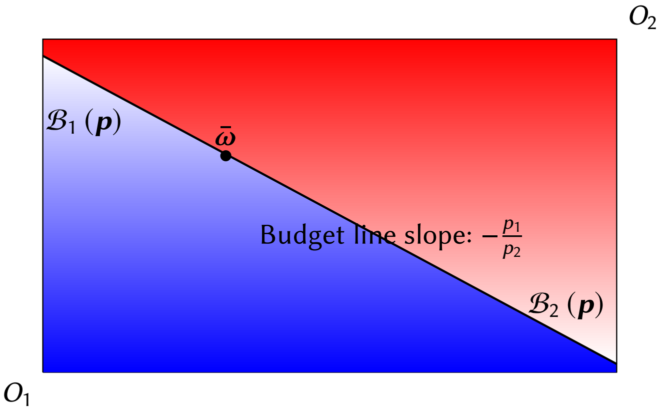
Edgeworth Box: Allocations

The point $\boldsymbol{x}$ describes each consumer's consumption of each good (under no disposal):



Edgeworth Box: Budget Sets

A consumer's budget set is: $\mathcal{B}_i(\boldsymbol{p}) = \left\{\boldsymbol{x}_i \in \mathbb{R}^2_+ : \boldsymbol{p} \cdot \boldsymbol{x}_i \leq \boldsymbol{p} \cdot \boldsymbol{\omega}_i\right\}$

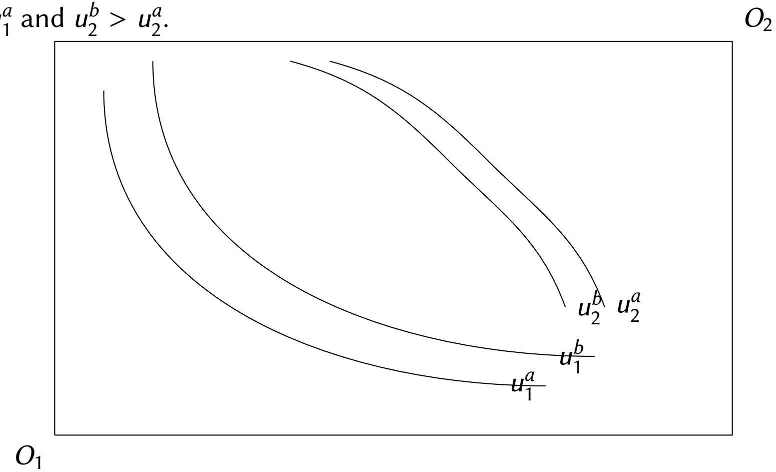


Only bundles on the budget line are affordable to both consumers simultaneously.

Edgeworth Box: Indifference Curves

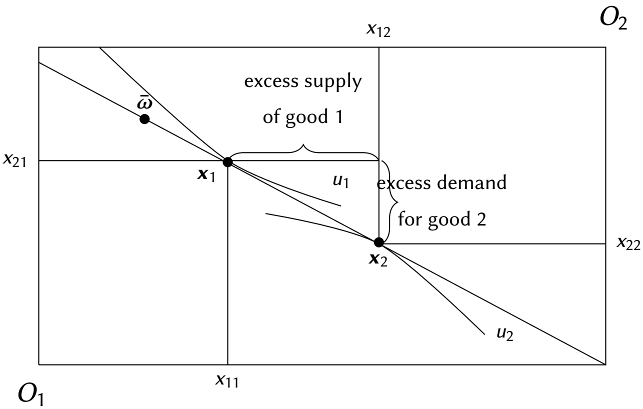
Example with strongly montone, continuous and strictly convex preferences:

- Consumer 1 prefers bundles towards the north east.
- Consumer 2 prefers bundles towards the south west.
- $u_1^b > u_1^a$ and $u_2^b > u_2^a$.



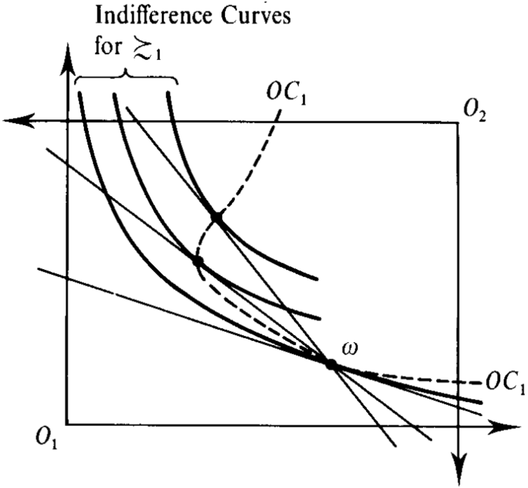
Edgeworth Box: Demand

- Consumer 1 is a net demander of good 1 and a net supplier of good 2.
- Consumer 2 is a net supplier of good 1 and a net demander of good 2.
- However, markets do not clear at these prices, as $x_{11} + x_{12} < \bar{\omega}_1$ and $x_{21} + x_{22} > \bar{\omega}_2$.



Edgeworth Box: Offer Curve

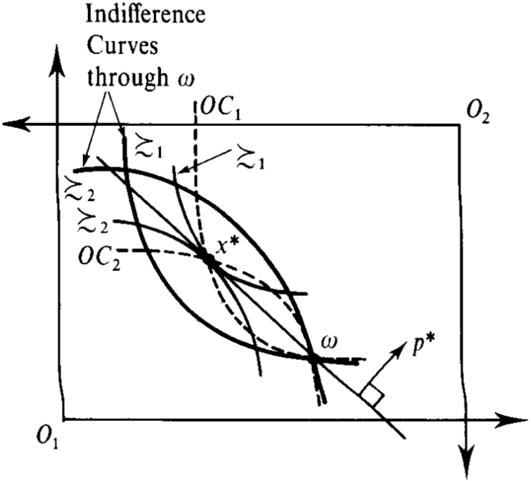
A consumer's *offer curve* traces out the consumer's demand at each price vector $\boldsymbol{p}$. Since $\boldsymbol{\omega}_i$ is always affordable, it lies in the upper contour set of $\boldsymbol{\omega}_i$.



Source: Mas-Colell, A., et al. (1995) *Microeconomic Theory*

Edgeworth Box: Intersection of Offer Curves

When both consumers' offer curves intersect, the total amount demanded equals the total endowment for each good: the market clears.

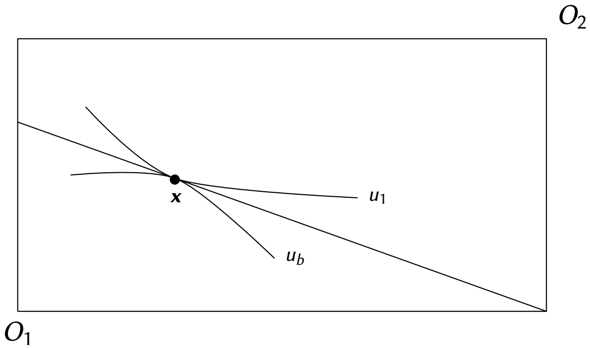


Source: Mas-Colell, A., et al. (1995) *Microeconomic Theory*

Edgeworth Box: Equilibrium

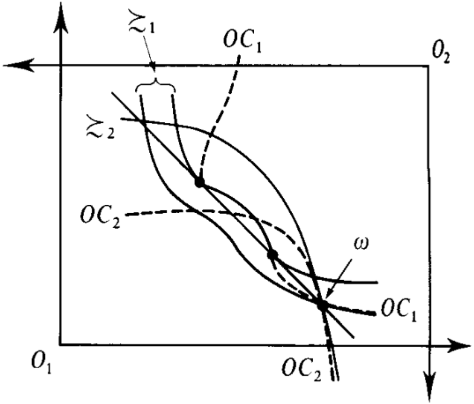
Definition

A *Walrasian equilibrium* for an Edgeworth box economy is a price vector $\boldsymbol{p}^\star$ and an allocation $\boldsymbol{x}^\star = \left(\boldsymbol{x}_1^\star, \boldsymbol{x}_2^\star\right)$ in the Edgeworth box such that for $i = 1, 2$, $\boldsymbol{x}_i^\star \succeq_i \boldsymbol{x}_i'$ for all $\boldsymbol{x}_i' \in \mathcal{B}_i\left(\boldsymbol{p}^\star\right)$.



Nonexistence of Equilibria: Nonconvex Preferences

- Equilibria do not always exist:



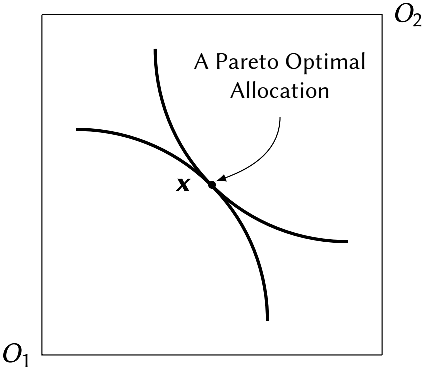
Source: Mas-Colell, A., et al. (1995) *Microeconomic Theory*

- The consumers' offer curves never intersect at any point where $\boldsymbol{x}_i \neq \boldsymbol{\omega}_i$.
- $\boldsymbol{x}_i = \boldsymbol{\omega}_i$ is also not an equilibrium.

Pareto Optimality

Definition

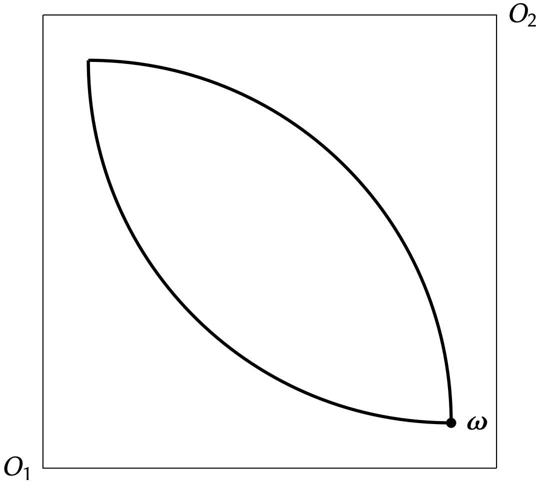
An allocation $\boldsymbol{x}$ in the Edgeworth box is *Pareto optimal* if there is no other allocation $\boldsymbol{x}'$ in the Edgeworth box with $\boldsymbol{x}'_i \succeq_i \boldsymbol{x}_i$ for $i = 1, 2$ and $\boldsymbol{x}'_i \succ \boldsymbol{x}_i$ for some i.



With smooth indifference curves, interior Pareto optimal allocations occur at the tangency.

The Lens of Pareto Improvements on $\boldsymbol{\omega}$

$$\left\{(\boldsymbol{x}_1, \boldsymbol{x}_2) \in \mathbb{R}^4_+ : \boldsymbol{x}_1 \succeq_1 \boldsymbol{\omega}_1 \text{ and } \boldsymbol{x}_2 \succeq_2 \boldsymbol{\omega}_2 \text{ and } \boldsymbol{x}_1 + \boldsymbol{x}_2 = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2\right\}$$



The interior of the lens are all Pareto improvements on $\boldsymbol{\omega}$.

The Pareto Set

- The set of Pareto optimal allocations is called the Pareto set.
- In the pure exchange Edgeworth box, the Pareto set is:

$$\mathcal{P} = \Big\{ (\boldsymbol{x}_1, \boldsymbol{x}_2) \in \mathbb{R}^4_+ : \nexists\, \boldsymbol{x}'_1, \boldsymbol{x}'_2 \text{ satisfying } \boldsymbol{x}'_1 + \boldsymbol{x}'_2 \leq \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$$
$$\text{and } \boldsymbol{x}'_i \succeq_i \boldsymbol{x}_i \, \forall i = 1, 2 \text{ and } \boldsymbol{x}'_i \succ_i \boldsymbol{x}_i \text{ for some } i \Big\}$$

- With well-behaved preferences, the union of the locus of tangencies of the indifference curves and the origins make up the Pareto set.

The Contract Curve

- The Pareto set is the red and blue line.
- The contract curve, CC, is a subset of the Pareto set where the allocations are at least as good as the endowment for each consumer (red line):

$$CC = \left\{(\boldsymbol{x}_1, \boldsymbol{x}_2) \in \mathbb{R}^4_+ : \boldsymbol{x}_1 \succeq_1 \boldsymbol{\omega}_1 \text{ and } \boldsymbol{x}_2 \succeq_2 \boldsymbol{\omega}_2\right\} \cap \mathcal{P}$$