Externalities and Public Goods

230333 Microeconomics 3 (CentER) – Part II
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Introduction

- Here we consider two types of *market failures*, where competitive equilibria may fail to be Pareto efficient.
- If a consumer's preferences depends on the consumption of other consumers and/or a firm's output depends on other firms' production, there may be *externalities*.
  - These consumers and firms do not internalize the preferences of others, and thus competitive equilibria may not be efficient.
- Non-rival and non-excludable goods are called *public goods*.
  - Under private provision, these will be underprovided due to the *free-rider problem*.
- Rival and non-excludable goods are *common goods*.
  - This can lead to the *tragedy of the commons* (to be studied in the assignment).

Definition of an Externality

Definition

An *externality* is present whenever the well-being of a consumer or the production possibilities of a firm are directly affected by the actions of another agent in the economy.

Examples:

- Positive consumption externality: Getting the flu vaccine.
- Negative consumption externality: Smoking in a restaurant.
- Positive production externality: Apiculture (on horticulture).
- Negative production externality: Pollution from a factory.

Simple Bilateral Externalities

- We consider a partial equilibrium model and two consumers out of the overall economy with wealth levels w_i, $i = 1, 2$.
- Each consumer has preferences over the L traded goods *and* some action $h \in \mathbb{R}_+$ taken by consumer 1.
- Consumer i's utility function is $u_i(\mathbf{x}_i, h)$ where $\frac{\partial u_2(\mathbf{x}_i, h)}{\partial h} \neq 0$.
- If $\frac{\partial u_2(\mathbf{x}_2, h)}{\partial h} > 0$, we have a positive consumption externality.
- If $\frac{\partial u_2(\mathbf{x}_2, h)}{\partial h} < 0$, we have a negative consumption externality.

Simple Bilateral Externalities

- We assume utility $u_i(\boldsymbol{x}_i, h) = g_i(\boldsymbol{x}_{-1i}, h) + x_{i1}$ is quasilinear with respect to the numeraire commodity (good 1).
- Demand for goods $\ell > 1$ is then $\boldsymbol{x}_{-1i}(\boldsymbol{p}, h)$, which is independent of wealth w_i.
- Define:

$$\phi_i(\boldsymbol{p}, h) = g_i(\boldsymbol{x}_{-1i}(\boldsymbol{p}, h), h) - \boldsymbol{p} \cdot \boldsymbol{x}_{-1i}(\boldsymbol{p}, h)$$

- Then each consumer's indirect utility function takes the form:

$$\begin{aligned} v_i(\boldsymbol{p}, w_i, h) &= g_i(\boldsymbol{x}_{-1i}(\boldsymbol{p}, h), h) + x_{i1}(\boldsymbol{p}, w_i) \\ &= g_i(\boldsymbol{x}_{-1i}(\boldsymbol{p}, h), h) + w_i - \boldsymbol{p} \cdot \boldsymbol{x}_{-1i}(\boldsymbol{p}, h) \\ &= \phi_i(\boldsymbol{p}, h) + w_i \end{aligned}$$

- We will suppress the argument $\boldsymbol{p}$ because we only consider changes in h.

Competitive Outcome

- $\phi_2'(h) > 0$ if positive externality
- $\phi_2'(h) < 0$ if negative externality.
- Assume $\phi_i''(h) < 0$ for $i = 1, 2$.
- Consumer 1's problem is:

$$\max_{h \geq 0} \phi_1(h) + w_1$$

- In a competitive outcome, consumer 1 optimally sets $h^\star$ such that $\phi_1'\left(h^\star\right) \leq 0$, with equality if $h^\star > 0$.

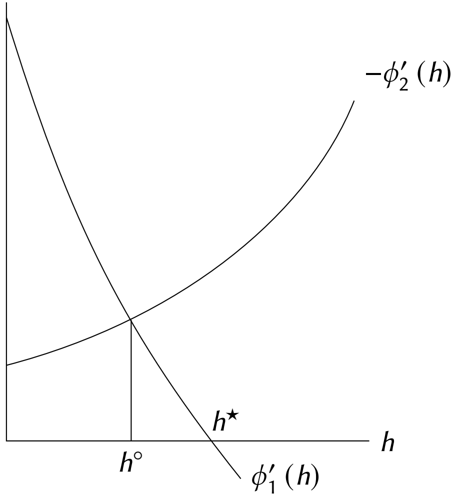
Non-Optimality of the Competitive Outcome

- A social planner solves:

$$\max_{h \geq 0} \ \phi_1(h) + \phi_2(h)$$

- The first-order condition is $\phi_1'(h^\circ) \leq -\phi_2'(h^\circ)$, with equality if $h^\circ > 0$.
- In the competitive outcome, $h^\star$ is such that $\phi_1'\left(h^\star\right) \leq 0$, with equality if $h^\star > 0$.
- When externalties are present ($\phi_2'(h) \neq 0$), the competitive outcome is not optimal unless $h^\star = h^\circ = 0$.
- In interior solutions:
    - If there is a negative externality: $h^\star > h^\circ$.
    - If there is a positive externality: $h^\star < h^\circ$.

Negative Externality



Pigouvian Taxation On Negative Externalities

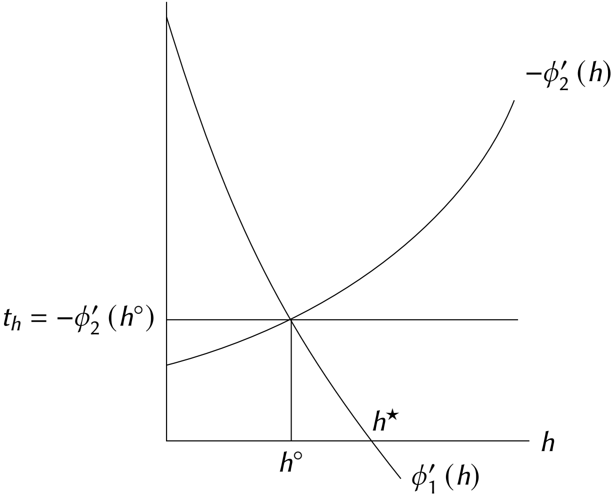
- Suppose there is a tax t_h on each unit of h.
- Consumer 1's problem becomes:

$$\max_{h \geq 0} \ \phi_1(h) - t_h h$$

- This has the FOC: $\phi_1'(h) \leq t_h$, with equality if $h > 0$.
- Setting a tax $t_h = -\phi_2'(h^\circ)$ will achieve the optimal outcome.
- This tax makes consumer 1 internalize the externality.

Negative Externality

Setting a tax $t_h = -\phi_2'(h^\circ)$ will achieve the optimal outcome:



Pigouvian Taxation On Negative Externalities

- The same result could be achieved by instead by offering a per-unit subsidy for the reduction of h below $h^{\star}$.
- The consumer's problem is then:

$$\phi_1(h) + s_h\left(h^{\star} - h\right)$$

- This has the FOC: $\phi_1'(h) \leq s_h$, with equality if $h > 0$.
- $s_h = -\phi_2'(h^{\circ})$ will result in the optimal outcome.
- This is equivalent to a tax on h with a lump sum transfer $s_h h^{\star}$.

Coase Theorem

- In reality, Pigouvian taxation requires a lot of information.
  - It requires knowledge of all consumers' utility functions with respect to the externality.

Coase Theorem

If trade of the externality can occur, then bargaining will lead to an efficient outcome, no matter how property rights are allocated.

- Suppose $\phi_2'(h) < 0$ and we give consumer 2 enforceable property rights for an externality-free environment.
- Consumer 1 must bargain with consumer 2 to be able to consume h.

Coase Theorem

- Assume bargaining is a take-it-or-leave-it offer where consumer 1 pays T in return for the right to consume h units.
- Consumer 2's problem is then:

$$\max_{h \geq 0, T} \; \phi_2(h) + T \text{ subject to } \phi_1(h) - T \geq \phi_1(0)$$

- At the optimum, the constraint binds. Substituting yields the social planner's problem:

$$\max_{h \geq 0} \; \phi_2(h) + \phi_1(h) - \phi_1(0)$$

- If consumer 1 owns the property rights, the same outcome will occur, with instead consumer 2 paying consumer 1 to consume less.
- The distribution of surplus depends on the distribution of property rights and the bargaining procedure.

Public Goods

Definition

A *public good* is a commodity for which use of a unit of the good by one agent does not preclude its use by other agents.

- A public good is *nondepletable* and *nonexcludable*.
- Consider again a market with L traded, private goods and one public good.
- Assume the price or quantity of the public good doesn't affect prices of the other goods (partial equilibrium).
- The derived utility for consumer i is then $\phi_i(x)$, where $\phi_i'(x) > 0$ and $\phi_i''(x) < 0$, $\forall i$ and $\forall x \geq 0$.
- The cost of supplying the public good is $c(q)$, with $c'(q) > 0$ and $c''(q) > 0$, $\forall q \geq 0$.

Pareto Optimal Allocation of the Public Good

- The social planner solves:

$$\max_{q \geq 0} \sum_{i=1}^{I} \phi_i(q) - c(q)$$

- The FOC is $\sum_{i=1}^{I} \phi_i'(q^\circ) \leq c'(q^\circ)$ with equality if $q^\circ > 0$.
- In an interior solution, the sum of the marginal benefits equals the marginal cost.

Inefficiency of Private Provision of the Public Good

- Suppose the equilibrium price for the public good is $p^\star$.
- Given $p^\star$, each individual solves:

$$\max_{x_i \geq 0} \; \phi_i \left(x_i + \sum_{k \neq i} x_k^\star \right) - p^\star x_i$$

- The FOC is $\phi_i' \left(x_i^\star + \sum_{k \neq i} x_k^\star \right) \leq p^\star$, with equality if $x_i^\star > 0$.
- The firm's FOC is $p^\star \leq c' \left(q^\star \right)$, with equality if $q^\star > 0$.
- In equilibrium, markets clear so $\sum_{i=1}^{I} x_i^\star = q^\star$

Inefficiency of Private Provision of the Public Good

- The equilibrium conditions again are:
  - $\phi_i'\left(x_i^\star + \sum_{k\neq i} x_k^\star\right) \leq p^\star$, with equality if $x_i^\star > 0$.
  - $p^\star \leq c'\left(q^\star\right)$, with equality if $q^\star > 0$.
  - $\sum_{i=1}^{I} x_i^\star = q^\star$.
- Therefore we can write:

$$\sum_{i=1}^{I} \mathbb{1}\left\{x_i^\star > 0\right\}\left[\phi_i'\left(q^\star\right) - c'\left(q^\star\right)\right] = 0$$

- If $I > 1$ and if $x_i^\star > 0$ for at least one i:

$$\sum_{i=1}^{I} \phi_i'\left(q^\star\right) > c'\left(q^\star\right)$$

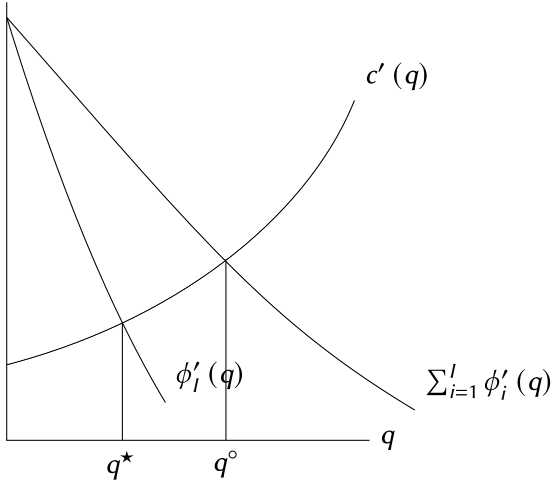
- If $q^\circ > 0$, then the planner sets $\sum_{i=1}^{I} \phi_i'\left(q^\circ\right) = c'\left(q^\circ\right)$.
- Since $\sum_{i=1}^{I} \phi_i'\left(q\right) - c'\left(q\right)$ is strictly decreasing in q, $q^\star < q^\circ$.
- There is underprovision of the public good.

Free-Rider Problem

- The consumption of the public good by one consumer has a positive externality on other consumers.
- Each consumer's incentive is to enjoy the benefits provided by others but not provide it themselves.
  - This is the *free-rider problem.*
- Suppose $\phi_1'(x) < \cdots < \phi_I'(x), \forall x \geq 0$.
- Then the FOC can only hold with equality for one consumer.
- $q^\star$ in this case satisfies $\phi_I'\left(q^\star\right) = c'\left(q^\star\right)$ if $\phi_I'(0) > c'(0)$.

Free-Rider Problem

If $\phi_1'(x) < \cdots < \phi_I'(x), \forall x \geq 0$ and $\phi_I'(0) > c'(0)$:



Solving the Free-Rider Problem

- Typically public goods are instead provided by the government.
- A price-based solution involves subsidizing the purchase of the public good with $s_i = \sum_{k \neq i} \phi_k' (q^\circ)$ for each i.
- Consumer i's problem becomes:

$$\max_{x_i \geq 0} \phi_i \left(x_i + \sum_{k \neq i} x_k \right) + \underbrace{\sum_{k \neq i} \phi_k' (q^\circ)}_{= s_i} x_i - p x_i$$

which, if $q^\circ > 0$, has first-order conditions:

$$\phi_i' (q^\star) + \sum_{k \neq i} \phi_k' (q^\circ) - \underbrace{p}_{= c' (q^\star)} = 0$$

- This is the same as from the planner's problem, so $q^\star = q^\circ$.

Multilateral Externalities

- Often externalities are felt by multiple parties.
- We will study two types:
  - *Depletable externalities*: one consumer's experience of the externality reduces the amount needed to be felt by others.
    - For example, dumping waste on private property.
  - *Nondepletable externalities*: All consumers experience the same total level of the externality.
    - For example, air pollution.

Multilateral Externalities

- There are J firms generating the externalities and I consumers.
- A firm's derived profit function as a function of its generated externality $h_j \geq 0$ is $\pi_j\left(h_j\right)$ with $\pi_j''\left(h_j\right) < 0$.
- Firms choose $h_j^\star$ satisfying $\pi_j'\left(h_j^\star\right) \leq 0$, with equality if $h_j^\star > 0$.
- Consumers have quasilinear preferences over the L goods and their perceived externality.
- A consumer's derived utility function is $\phi_i\left(\widetilde{h}_i\right)$, with $\phi_i''\left(\widetilde{h}_i\right) < 0$.
- With depletable externalities, $\sum_{i=1}^{I} \widetilde{h}_i = \sum_{j=1}^{J} h_j$.
- With nondepletable externalities, $\widetilde{h}_i = \sum_{j=1}^{J} h_j$ for all i.

Depletable Externalities

- The Pareto efficient allocation is the argument that solves:

$$\max_{\substack{(h_1,\ldots,h_J)\geq\mathbf{0},\\ (\widetilde{h}_1,\ldots,\widetilde{h}_I)\geq\mathbf{0}}} \sum_{i=1}^{I}\phi_i\left(\widetilde{h}_i\right)+\sum_{j=1}^{J}\pi_j\left(h_j\right) \text{ subject to } \sum_{j=1}^{J}h_j=\sum_{i=1}^{I}\widetilde{h}_i$$

- The FOCs are $\phi_i'\left(\widetilde{h}_i^\circ\right)\leq\mu$, with equality if $\widetilde{h}_i^\circ>0$, $\forall i$ and $\mu\leq-\pi_j'\left(h_j^\circ\right)$, with equality if $h_j^\circ>0$, $\forall j$.
- These are analogous to the conditions in the First Welfare Theorem:
  - If markets are competitive and property rights over the externality are well-defined and enforceable, then the Pareto optimal allocation can be achieved.

Nondepletable Externalities

- With nondepletable externalities, the Pareto efficient allocation is the maximizer in the problem:

$$\max_{(h_1,\dots,h_J)\geq \mathbf{0}} \; \sum_{i=1}^{I} \phi_i \left(\sum_{j=1}^{J} h_j \right) + \sum_{j=1}^{J} \pi_j \left(h_j \right)$$

- The FOCs are $\sum_{i=1}^{I} \phi_i' \left(\sum_{j=1}^{J} h_j^\circ \right) \leq -\pi_j' \left(h_j^\circ \right)$, with equality if $h_j^\circ > 0$, for all j.
- In the competitive outcome, each firm chooses $h_j^\star$ to satisfy $\pi_j' \left(h_j^\star \right) \leq 0$, with equality if $h_j^\star > 0$.
- With a Pigouvian tax of $t_h = -\sum_{i=1}^{I} \phi_i' \left(\sum_{j=1}^{J} h_j^\circ \right)$ on each unit of the externality, however, we can achieve the Pareto optimal outcome.

Tradeable Permits

- The social planner can also achieve the optimal level of the externality $h^\circ = \sum_{j=1}^{J} h_j^\circ$ with tradeable permits.
- The social planner distributes permits to each firm to produce up to $\bar{h}_j$ units of the externality, where $\sum_{j=1}^{J} \bar{h}_j = h^\circ$.
- Firms are allowed to buy and sell the externality in a centralized market at price $p_h^\star$.
- Each firm will choose h_j to solve $\max_{h_j \geq 0} \; \pi_j(h_j) - p_h^\star (h_j - \bar{h}_j)$.
- The FOCs are $\pi_j'\left(h_j^\star\right) \leq p_h^\star$, with equality if $h_j^\star > 0$.
- By market clearing, we have $\sum_{j=1}^{J} h_j^\star = h^\circ$.
- The equilibrium price will be $p_h^\star = -\sum_{i=1}^{I} \phi_i'(h^\circ)$.
- Here, the planner only needs to know h°, and not each individual's utility function or each firm's profit function.