Price Discrimination

Question 1 – Third-Degree Price Discrimination

There is one movie theater in a town. The town has two groups of people: students and non-students. The non-students have a demand curve:

$$D_1(p_1) = 10 - p_1$$

and the students have a demand curve:

$$D_2(p_2) = 10 - 2p_2$$

The movie theater has a marginal cost of \$2 per customer (cleaning up their spilled soda and popcorn). The fixed cost is 10.

(i) Suppose first the monopolist movie theater cannot price discriminate. What price should they charge and what quantity will it sell? What profit does it make?

The full demand curve is:

$$D(p) = D_1(p) + D_2(p) = 10 - p_1 + 10 - 2p = 20 - 3p$$

The inverse demand curve is then:

$$p = \frac{20}{3} - \frac{q}{3}$$

The revenue function is then:

$$R(q) = p(q)\,q = \frac{20}{3}q - \frac{1}{3}q^2$$

The marginal revenue is then:

$$MR(q) = \frac{20}{3} - \frac{2}{3}q$$

Setting $MR(q) = MC(q)$ gives:

$$\frac{20}{3} - \frac{2}{3}q = 2 \quad \Longrightarrow \quad q = 7$$

The price is $\frac{20}{3} - \frac{7}{3} = \frac{13}{3} = 4\frac{1}{3}$. The profit is then:

$$\pi = \frac{13}{3} \times 7 - 2 \times 7 - 10 = 6\frac{1}{3}$$

(ii) Suppose now the monopolist is able to price discriminate. What price should it charge students and what price should it charge non-students? How many tickets will they sell?
The individual inverse demand curves are:

$$p_1(q_1) = 10 - q_1$$

and:

$$p_2(q_2) = 5 - \frac{1}{2}q_2$$

The revenue and marginal revenue functions for each group are:

$$R_1(q_1) = 10q_1 - q_1^2 \qquad MR_1(q_1) = 10 - 2q_1$$

and:

$$R_2(q_2) = 5q_2 - \frac{1}{2}q_2^2 \qquad MR_2(q_2) = 5 - q_2$$

The monopolist's problem is then to set the marginal revenue from each group equal to marginal cost:

$$10 - 2q_1 = 2 \implies q_1 = 4$$
$$5 - q_2 = 2 \implies q_2 = 3$$

The prices are:

$$p_1 = 10 - 4 = 6 \qquad p_2 = 5 - \frac{3}{2} = 3\frac{1}{2}$$

The profit is then:

$$\pi = p_1 q_1 + p_2 q_2 - (q_1 + q_2)\, c - F = 6 \times 4 + \left(3\frac{1}{2}\right) \times 3 - (3 + 4) \times 2 - 10 = 10.5$$

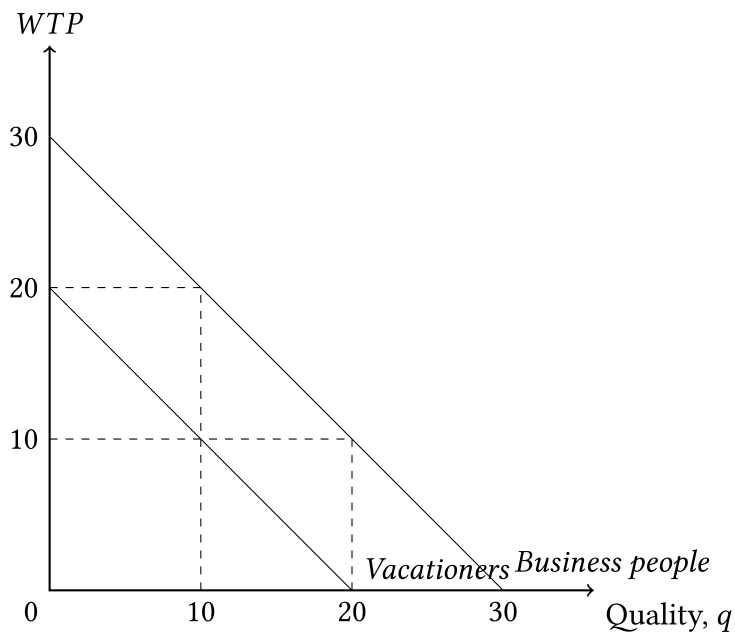
The firm makes a higher profit by by discriminiating.

Question 2 – Monopoly Price Discrimination

A monopolist airline faces two types of consumers who value flight quality differently. There is a high willingness-to-pay group (business people) and a low willingness-to-pay group (vacationers). The airline faces zero marginal cost (putting one more person on a plane doens't any extra).

"Quality" is measured on a 30-point scale, where 30 is extremely luxurious and 0 is extremely uncomfortable.

The two solid downward-sloping lines below measure the willingness-to-pay for a marginal increase in quality for both groups. For any quality level, q, the willingness-to-pay is the area *under the line* between between 0 and q. Each triangle blow has an area of 50 and each square has an area of 100. If $q = 10$, vacationers are willing to pay at most 150 for air travel, while business people would pay 250. If the quality level is 0, the area is always 0 so no one would pay for airfare at quality 0.



We can summarize the willingness to pay for each type for each quality level as follows:

	Vacationers	Business people
Low quality ($q = 10$)	150	250
Medium quality ($q = 20$)	200	400
High quality ($q = 30$)	200	450

(i) If the only people in the world were business people, what should the airline charge for air travel to maximize profits?

You should set the quality level to 30 and charge exactly what the market will value the flight. This is the area of the entire triangle which is 450.

(ii) If the only people in the world were vacationers, what should the airline charge for air travel to maximize profits?

You should set the quality level to 20 and charge exactly what the market will value the flight. This is the area of the smaller triangle which is 200.

For the rest of the question, assume that 50% of air travelers are business people and 50% are vacationers.

(iii) Suppose vacationers were able to obtain a "vacationer's card" which allowed them to get discounts for air travel if discounts were available (like a student card). The airline now has an observable way to tell travelers apart. What should the airline charge each group to maximize profits? What is the average profit made per person?[1]

[1] If you get revenue x from vactioners and revenue y from business people, the average profit per person is $\frac{x+y}{2}$. If you only sell to business people, the average profit per person is $\frac{y}{2}$.

The airline should just charge each group for the different quality levels in (i) and (ii). The vacationers wouldn't want to buy the high quality flight (they would get negative surplus by doing so) so they wouldn't pretend to be business people. Business people would want to go on the lower-quality flight, but since they don't have "vacationers" cards they can't get tickets for it. Therefore the airline offers a high quality flight with $q_H = 30$ for 450 and a medium-quality flight with $q_M = 20$ for 200. The average profit made per person is $0.5 \times 450 + 0.5 \times 200 = 325$.

(iv) Suppose now that a vacationer's card doesn't exist so the airline can't charge different people different prices. If the airline were to choose only one price-quality plan, what would it do to maximize profits? To do this you should check two possibilities:

- You set a price-quality plan such that both groups would be willing to buy.
- You set a price-quality plan such that only business people will be willing to buy (but you lose out on half the market).

Find the average profit made per person from both of these possibilities.

- If the airline wanted to sell to both groups it should offer $q = 20$ for 200. Vacationers would buy it and get zero surplus. The business people would buy it and get surplus equal to 200. The average profit per person is 200 (since everyone buys it).
- If the airline focused only on the business people, the airline should offer $q = 30$ like in (i) for 450. The average profit per person is then $0.5 \times 450 = 225$ (since you lose the vacationers).
- Therefore if the airline was only going to set a single price-quality pair, it should sell only to business people.

(v) Suppose again that the vacationer's card doesn't exist but the airline now wants to set two price-quality plans in order to make the vacationers and business people *self-select* into each plan.

- What price-quality plans for both groups would make the groups self-select and maximize profits?
- What is the average profit per customer?

Here we need to check the different combinations of quality levels and choose the one that maximizes profits.

We should also check the other combinations of qualities to confirm that this indeed is the profit-maximizing strategy:

- $q_L = 10$ and $q_M = 30$. If the airline tried to sell $q = 10$ to vacationers it would sell it for 150 (their willingness to pay). Business people would get 100 surplus from this so they need to give business people 100 surplus on the high quality flight. Their willingness to pay for the high quality flight is 450 so they should charge them 350 to ensure they are not tempted to buy the low quality flight. The average profit made per person is then $0.5 \times 150 + 0.5 \times 350 = 250$.

- $q_M = 20$ and $q_H = 30$. If the airline tried to sell $q = 20$ to vacationers it would sell it for 200. Business people would get 200 surplus from this so they should charge 450-200=250 for the $q_H = 30$ flight. On average this earns $0.5 \times 200 + 0.5 \times 250 = 225$ which is less than 250 from before.
- $q_L = 10$ and $q_M = 20$. If the airline tried to sell $q = 10$ to vacationers it would sell it for 150. Business people would get 100 surplus from this so they should charge 400-100=300 for the $q_H = 20$ flight. On average this earns $0.5 \times 150 + 0.5 \times 300 = 225$ which is less than 250 from before.

Therefore the best plan for the airline is to offer a low quality flight $q_L = 10$ for 150 and a high quality flight $q_H = 30$ for 450. The airline makes a higher profit with two different options than by offering only one, even though it can't tell the two groups of consumers apart.

Question 3 – Two-Part Tariffs

There is a single theme park in a large state. It can put people on a roller coaster at zero marginal cost (but they do have a fixed cost). The demand for roller coaster rides for each person is given by $D(p) = 200 - \frac{1}{2}p$, where p is the price of each roller coaster ride. What price should the theme park charge for entry into the theme park and how much should it charge for each person to ride the roller coaster?

We saw in class the optimal thing to do in a two-part tariff is to charge at marginal cost (here zero) and then have the entrance fee equal to the consumer surplus that the consumers would get if you only charged at marginal cost. The inverse demand curve is $p(q) = 400 - 2q$. The area of the triangle under this demand curve is $\frac{1}{2} \times 400 \times 200 = 40,000$ Therefore the theme park should charge an entry fee of $40,000 and charge nothing for the rides. This steals the entire consumer surplus from consumers and there is no deadweight loss.

Game Theory

Question 4 – Simulataneous Games

Find all Nash equilibria (pure and mixed) of the following games:

In the following, for mixed-strategy equilibria, p is the probability player 1 goes up (U) and q is the probability that player 2 goes left (L).

(i)

	L	R
U	2, 4	6, 3
D	3, 1	5, 3

There are no pure strategy equilibria. There is a mixed-strategy equilibrium with $p = \frac{2}{3}$ and $q = \frac{1}{2}$. To see how we get these probabilities, player 1 plays U with probability p and player 2 plays L with probability q. For player 1 to be indifferent between the two actions we need:

$$\begin{aligned}2q + 6(1-q) &= 3q + 5(1-q)\\ 2q + 6 - 6q &= 3q + 5 - 5q\\ 1 &= 2q\\ q &= \frac{1}{2}\end{aligned}$$

For player 2 to be indifferent between the two actions we need:

$$\begin{aligned}4p + (1-p) &= 3p + 3(1-p)\\ 4p + 1 - p &= 3\\ 3p &= 2\\ p &= \frac{2}{3}\end{aligned}$$

(ii)

	L	R
U	2, 1	0, 0
D	0, 0	1, 2

(U, L) and (D, R) are pure strategy equilibria. There is also a mixed strategy equilibrium with $p = \frac{2}{3}$ and $q = \frac{1}{3}$.

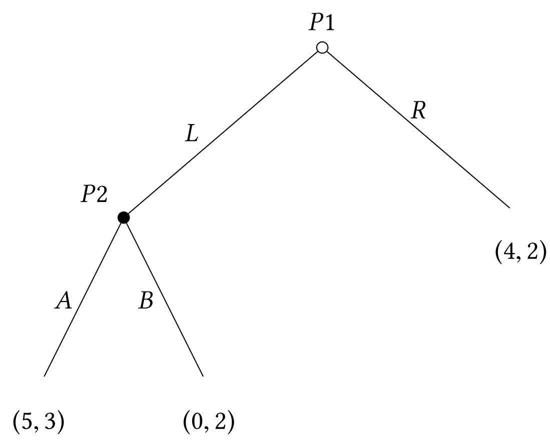
(iii)

	L	R
U	−2, −2	4, 0
D	0, 4	2, 2

(D, L) and (U, R) are pure strategy equilibria. There is also a mixed strategy equilibrium with $p = q = \frac{1}{2}$.

Question 5 – Sequential Games

Consider the following extensive form game:



(i) Write the normal form (payoff matrix) of the game.

The normal form is:

	A	B
L	5, 3	0, 2
R	4, 2	4, 2

(ii) Find all the Nash equilibria of the game.

There are two pure strategy NE: (R, B) and (L, A). There is also a MSNE where P1 does R always and P2 plays A with probability $\frac{4}{5}$ and B with probability $\frac{1}{5}$, i.e. $p = 0$ and $q = \frac{4}{4}$.

(iii) Which of the Nash equilibria that you found are subgame perfect?

(L, A) is the only SPNE.

Oligopoly

Question 6 – Different Market Structures

The demand curve for a good is given by $D(p) = 5 - \frac{1}{2}p$. The cost function for a firm is given by $c(q) = 2q$.

(i) The market was served by perfectly competitive firms. What is the market price and total output produced by the industry? What profit does each firm make?

In perfect competition, price equals marginal cost. The marginal cost here is $c'(q) = 2$. With $p = 2$ we can use the demand function to get $q = 5 - \frac{1}{2} \times 2 = 4$. Each firm makes zero profit since price equals marginal cost and there is no fixed cost.

(ii) The market is served by a monopolist. What price and quantity does the monopolist produce? What profit does it make?

The inverse demand curve is $p(q) = 10 - 2q$. The revenue function is then $R(q) = 10q - 2q^2$. The marginal revenue function is then $MR(q) = R'(q) = 10 - 4q$. In monopoly, to maximize profits you set $MR(q) = MC(q)$ to find the optimal quantity. So:

$$10 - 4q = 2 \quad \Longleftrightarrow \quad q = 2$$

Note that this is half of the perfectly competitive quantity. The price then is $p = 10 - 2 \times 2 = 6$. The monopolist's profits is then $\pi = (p - AC)\, q = (6 - 2)\, 2 = 8$.

(iii) The market is served by two firms who compete simultaneously on price. What price does each firm charge? What quantity is produced by both firms? What profit does each firm make?

This is Bertrand competition. If two firms compete on price then they will engage in a price war and end up charging $p = MC$, like in perfect competition. So $p = 2$, the quantity produced in total will be $q = 4$ and both firms will make zero profits.

(iv) The market is served by two firms who simulatenously choose their quantites. The price is then set to clear the market. What quantity is produced by both firms? What is the price? What is the profit for each firm?

This is Cournot competition. In general terms, firm 1's problem is:

$$\max_{q_1} p(q_1 + q_2)\, q_1 - c(q_1)$$

The revenue term is:

$$p(q_1 + q_2)\, q_1 = [10 - 2(q_1 + q_2)]\, q_1 = 10q_1 - 2q_1^2 - 2q_1 q_2$$

Therefore firm 1's problem is:

$$\max_{q_1} 10q_1 - 2q_1^2 - 2q_1 q_2 - 2q_1$$

The first-order condition is then:

$$10 - 4q_1 - 2q_2 - 2 = 0$$

So $q_1(q_2) = 2 - \frac{1}{2}q_2$. This is firm 1's best response function. For firm 2, we can do the same steps and arrive at $q_2(q_1) = 2 - \frac{1}{2}q_1$. In equilibrium, both equations will be satisfied:

$$\begin{aligned} q_1 &= 2 - \frac{1}{2}q_2 \\ q_2 &= 2 - \frac{1}{2}q_1 \end{aligned}$$

Inserting the equation for q_2 into the equation for q_1:

$$\begin{aligned} q_1 &= 2 - \frac{1}{2}\left(2 - \frac{1}{2}q_1\right) \\ q_1 &= 2 - 1 + \frac{1}{4}q_1 \\ \frac{3}{4}q_1 &= 1 \\ q_1 &= \frac{4}{3} \end{aligned}$$

Using this in the best response function for firm 2 we get: $q_2 = 2 - \frac{1}{2} \times \frac{4}{3} = 2 - \frac{4}{6} = \frac{4}{3}$ also. Each firm produces one third of the perfectly competitive quantity. The industry as a whole then produces $q = 2 \times \frac{4}{3} = \frac{8}{3}$. This is two thirds of the perfectly competitive quantity. The price is then $p = 10 - 2 \times \frac{8}{3} = \frac{14}{3} = 4\frac{2}{3}$. Each firm makes profits $\pi = (p - AC)\,q = \left(\frac{14}{3} - 2\right)\frac{4}{3} = \frac{8}{3} \times \frac{4}{3} = \frac{32}{9} = 3\frac{5}{9}$.

(v) The market is served by two firms but one firm chooses the quantity to produce first. The other firm then observes this quantity and chooses its own quantity. The price is then set to clear the market. What quantity does each firm produce? What is the price? What profit does each firm make?

First we need to find how the follower will react to the leader's choice of quantity. This is exactly how the Cournot player's react to each other. Therefore firm 2's reaction function is:

$$q_2(q_1) = 2 - \frac{1}{2}q_1$$

Firm 1 then will take how firm 2 will react into account when deciding on a quantity. Using this expression in firm 1's profit function:

$$\begin{aligned} \pi_1(q_1) &= 10q_1 - 2q_1^2 - 2q_1q_2 - 2q_1 \\ \pi_1(q_1) &= 8q_1 - 2q_1^2 - 2q_1\left(2 - \frac{1}{2}q_1\right) \\ \pi_1(q_1) &= 8q_1 - 2q_1^2 - 4q_1 - q_1^2 \\ \pi_1(q_1) &= 4q_1 - q_1^2 \end{aligned}$$

Taking the derivative of this and setting equal to zero:

$$4 - 2q_1 = 0 \iff q_1 = 2$$

Note that this is the same as a monopolist quantity. The follower then will produce:

$$q_2 = 2 - \frac{1}{2}q_1 = 2 - \frac{1}{2} \times 2 = 1$$

This is one quarter of the perfectly competitive quantity. The industry as a whole then produces $2 + 1 = 3$. This is three quarters of the perfectly competitive quantity. The price is then $p = 10 - 2 \times 3 = 4$. The profit for the leader is $\pi_1 = (4 - 2) \times 2 = 4$. The profit for the follower is $\pi_2 = (4 - 2) \times 1 = 2$.

In summary, the answer for each sub-part of this question is:

	Industry Quantity	Price	Profit for each firm
Perfect competition	4	2	0
Monopoly	2	6	8
Bertrand	4	2	0
Cournot	$2\frac{2}{3}$ (where $q_1 = q_2 = 1\frac{1}{3}$)	$4\frac{2}{3}$	$3\frac{5}{9}$
Stackelberg	3 (where $q_1 = 2$, $q_2 = 1$)	4	$\pi_1 = 4$, $\pi_2 = 2$