Cost Curves

Question 1 – Cost Curves

The cost function of a firm is:

$$c(y) = \frac{1}{2}y^2 + 2$$

(i) What is the firm's variable cost function $c_v(y)$?

The variable cost is the portion of the cost function that varies with output. These are the terms that include y terms. Here this is just $\frac{1}{2}y^2$. So $c_v(y) = \frac{1}{2}y^2$.

(ii) What is the firm's fixed costs?

The fixed cost is the portion of the cost function that doesn't vary with output. This is the constant term in the cost function. Here that is 2. So $F = 2$.

(iii) What is the firm's average cost function?

$$AC(y) = \frac{c(y)}{y} = \frac{\frac{1}{2}y^2 + 2}{y} = \frac{\frac{1}{2}y^2}{y} + \frac{2}{y} = \frac{y}{2} + \frac{2}{y}$$

(iv) What is the firm's average variable cost function?

$$AVC(y) = \frac{c_v(y)}{y} = \frac{\frac{1}{2}y^2}{y} = \frac{1}{2}y$$

(v) What is the firm's average fixed cost function?

$$AFC(y) = \frac{F}{y} = \frac{2}{y}$$

(vi) What is the firm's marginal cost function?

$$MC(y) = \frac{dc(y)}{dy} = 2 \cdot \frac{1}{2}y^{2-1} + 0 = y$$

(vii) Verify that the marginal cost curve instersects the average cost curve when the average cost curve is at its minimum. That is, first find the minimum of $AC(y)$. Then secondly find y that solves $MC(y) = AC(y)$.

The minimum of $AC(y)$ happens when its derivative equals zero:

$$\frac{d(AC(y))}{dy} = \frac{1}{2} - \frac{2}{y^2}$$

Finding the y where this equals zero:

$$\frac{1}{2} - \frac{2}{y^2} = 0 \iff \frac{1}{2} = \frac{2}{y^2} \iff y^2 = 4 \iff y = 2$$

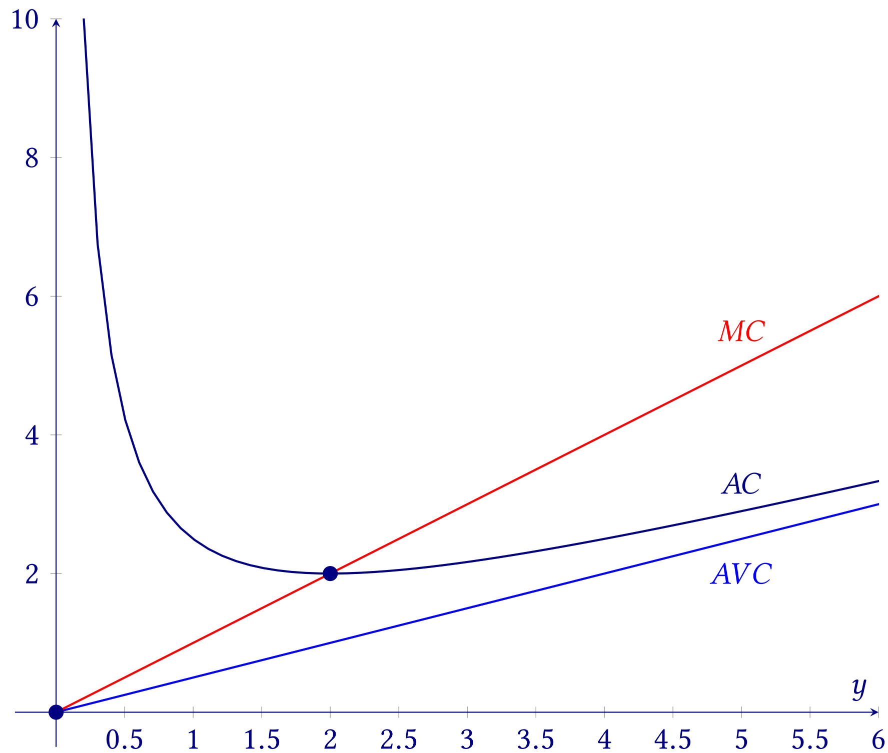
Therefore $AC(y)$ is at its minimum when $y = 2$[1]. If we set $MC(y) = AC(y)$ we get:

$$y = \frac{y}{2} + \frac{2}{y} \iff \frac{y}{2} = \frac{2}{y} \iff y^2 = 4 \iff y = 2$$

This confirms that $MC(y)$ and $AC(y)$ intersect at the minimum point of $AC(y)$ in this case.

(viii) Sketch the marginal cost, average cost and average variable cost function on a graph. Label the intersections of the curves with their values on the axes.

The minimum of $AC(y)$ happens at $(2, 2)$ and the minimum of $AVC(y)$ happens at $(0, 0)$.

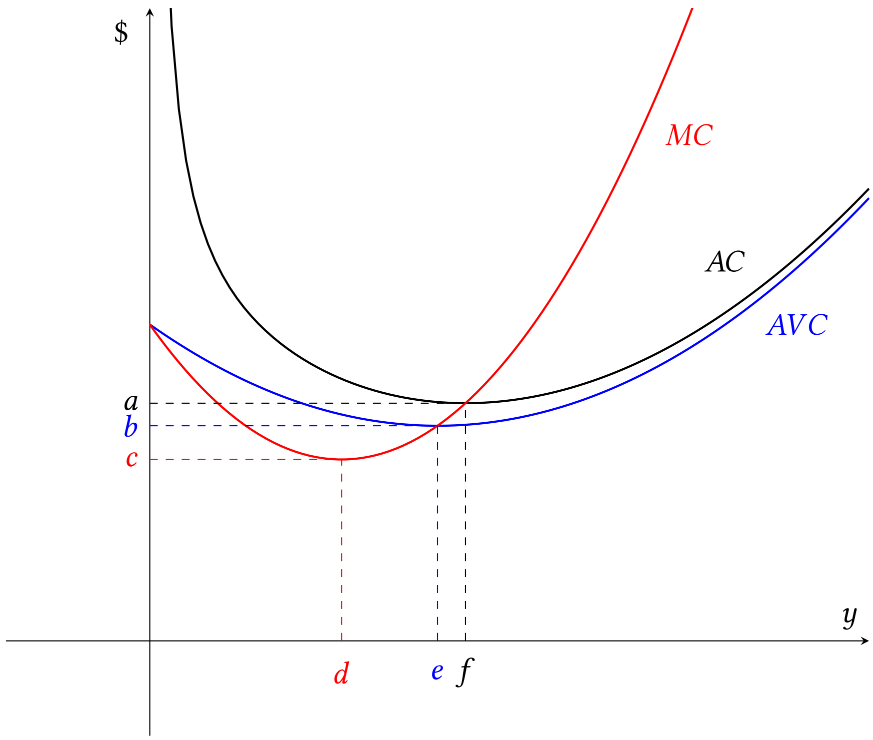


[1]Note that normally $y^2 = 4$ implies $y = \pm 2$ but since output is necessarily positive, we ignore the negative solution.

Firm Supply

Question 2 – Short-Run and Long-Run Supply

Consider the following marginal cost curve (MC), average cost curve (AC) and average variable cost curve (AVC).



The firm is in the **short run**:

(i) If $p = a$, how much will the firm produce (in terms of d, e and f)?

The firm produces the output where $p = MC(y)$. Thus the firm will produce f units.

(ii) If $p = a$, what will the firm's profit be (in terms of a, b, c, d, e and f)?

The firm's profit is $[p - AC(y)]\, y$. Since $p = AC(y)$ the firm will make a zero profit.

(iii) If $p = b$, how much will the firm produce (in terms of d, e and f)?

The firm produces the output where $p = MC(y)$. Thus the firm will produce e units. Here $p = AVC(y)$ so the firm is actually indifferent between temporarily shutting down and producing e.

(iv) If $p = b$, will the firm be making a profit or a loss?

Here $p < AC(y)$ so the firm will be making a loss.

(v) If $p = c$, how much will the firm produce (in terms of d, e and f)?

The firm will temporarily shut down if the price is less than average variable cost. This is the case here so the firm will produce 0 units of output.

Now the firm is in the **long run**:

(vi) If $p < a$, what will the firm do and why?

The firm will be making a loss if $p < a$ and would therefore exit the industry in the long run.

Industry Supply

Question 3 – Industry Supply

Consider a competitive industry where every firm has the same cost function $c(y) = \frac{1}{2}y^2 + 1$. The market demand function is $D(p) = 220 - 10p$.

(i) What is the supply function $S_i(p)$ for an individual firm in the industry? Hint: use the firm's optimality condition, $p = MC(y)$.

The marginal cost curve for the firm is $c'(y) = y$. Using $p = MC(y)$ we get $p = y$. The supply function for one firm is then $S_i(p) = p$.

(ii) If there are 100 firms in the industry, what will the industry supply curve, $S(p)$, be?

The industry supply curve just adds up all the individual supply curves. This is:

$$S(p) = S_1(p) + S_2(p) + \cdots + S_{100}(p) = p + p + \cdots + p = 100p$$

(iii) What will the (short-run) equilibrium price be?

The equilibrium price comes from setting supply equal to demand:

$$S(p) = D(p) \iff 100p = 220 - 10p \iff 110p = 220 \iff p = 2$$

(iv) How many units does the industry as a whole produce?

Using $p = 2$ in the industry supply function:

$$S(p) = 100p = 100 \times 2 = 200$$

The industry as a whole produces 200 units.

(v) How many units does a single firm produce?

Each firm produces output according to their supply function $S_i(p) = p = 2$. The 100 firms share the 200 units of output equally and produce 2 units.

(vi) What profit does each firm make?

The profit for each firm:

$$py - \frac{1}{2}y^2 - 1 = 2 \times 2 - \frac{1}{2} \times 2^2 - 1 = 1$$

The firms are making profits of 1.

(vii) Will any firms want to enter or exit this industry in the long run?

In the long run firms will want to enter this industry since they are making profits. Eventually, after enough firms enter. The price will fall until they break even

Equilibrium

Question 4 – Taxation

The demand and supply functions for a particular good in the market are given by:

$$D(p) = 12 - 2p$$
$$S(p) = 2 + 3p$$

(i) Find the equilibrium price and quantity.

To find the equilibrium price, $p^\star$, we set $D(p^\star) = S(p^\star)$:

$$12-2p^\star = 2+3p^\star \iff 12-2 = 2p^\star+3p^\star \iff 10 = 5p^\star \iff p^\star = \frac{10}{5} \iff p^\star = 2$$

The quantity can be found by using $p^\star$ in either $S(p)$ or $D(p)$. Using demand:

$$q^\star = 12 - 2 \times 2 = 8$$

Using supply:

$$q^\star = 2 + 3 \times 2 = 8$$

Now the government imposes a per-unit tax (a quantity tax) of 1 on the good.

(ii) Find the price the buyers pay, the price sellers receive and the new equilibrium quantity.

Now $P_D = P_S + t$. We can replace p in the demand function with $P_S + t$ and then set demand equal to supply:

$$\begin{aligned} 12 - 2(P_S + t) &= 2 + 3P_S \\ 12 - 2P_S - 2t &= 2 + 3P_S \\ 10 - 2t &= 5P_S \\ 5P_S &= 10 - 2t \\ P_S &= 2 - \frac{2}{5}t \\ P_S &= 2 - \frac{2}{5} \\ P_S &= 1.6 \end{aligned}$$

Using $P_D = P_S + t$ we can find the price buyers pay:

$$P_D = 2 - \frac{2}{5}t + t = 2 + \frac{3}{5}t = 2.6$$

The price increased for buyers by $\frac{3}{5}t = 60¢$ and the price decreased for sellers by $\frac{2}{5}t = 40¢$.

The new equilibrium quantity can be found using either the inverse demand or supply function. Using demand:

$$q^\star = 12 - 2P_D = 12 - 2 \times 2.6 = 12 - 5.2 = 6.8$$

Using supply:

$$q^\star = 2 + 3P_S = 2 + 3 \times 1.6 = 2 + 4.8 = 6.8$$

The tax decreased the equilibrium quantity by 1.2 units.

(iii) What portion of the tax do consumers pay and what portion of the tax do sellers pay?

Since the price increased by $\frac{3}{5}$ of the tax for consumers, they pay 60% of the tax. The price decreased by $\frac{2}{5}$ of the tax for sellers so they pay 40% of the tax.

Now suppose we are back to the situation **before the tax**. A recent policy change lowered income tax. This means that consumers now have more income. Suppose this means that all consumers are now willing to pay an extra $5 per unit of the good.

(iv) How does this affect the equilibrium price and quantity?

Recall from EC101 that income increases shift the demand curve to the right (for normal goods). An increase in the willingness to pay for the good by $5 for all consumers causes an upward parallel shift in the inverse demand curve. This means the intercept will increase by 5. The inverse demand curve is:

$$D(p) = 12 - 2p \implies 2p = 12 - q \implies P_D(q) = 6 - \frac{q}{2}$$

So the new inverse demand curve is (shifting the intercept up by 5):

$$P_D(q) = 11 - \frac{q}{2}$$

Translating this back to the demand curve is:

$$P_D(q) = 11 - \frac{q}{2} \implies 2p = 22 - q \implies D(p) = 22 - 2p$$

The supply curve stays the same. The equilibrium is found by setting $D(p) = S(p)$:

$$22 - 2p = 2 + 3p \iff 20 = 5p \iff p = 4$$

So the equilibrium price increases from 2 to 4. The equilibrium quantity is now:

$$22 - 2p^\star = 17 - 2 \times 2 = 14$$

So the equilibrium quantity increases from 8 to 14.