

MATHEMATICS REVIEW

Question 1 – Differentiation

Differentiate the following functions, i.e. find $\frac{df(x)}{dx}$:

(i) $f(x) = 4x^2$

(ii) $f(x) = 201$

(iii) $f(x) = 2x + 4x^3$

Question 2 – Partial Differentiation

Find the partial derivatives of the following functions, i.e. find $\frac{\partial f(x_1, x_2)}{\partial x_1}$ and $\frac{\partial f(x_1, x_2)}{\partial x_2}$:

(i) $f(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$

(ii) $f(x_1, x_2) = 2x_1 + 3x_2^2$

Question 3 – Optimization

For each of the functions below, do the following:

- Sketch the graph of the function.
- Find $\frac{df(x)}{dx}$
- Find the extreme point, i.e. at what x^* is $\frac{df(x^*)}{dx} = 0$?
- Is the extreme point a maximum or a minimum? (To do this you will need to find the second derivative.)

(i) $f(x) = 8x - 2x^2$

(ii) $f(x) = x^2$

THE BUDGET CONSTRAINT

Question 4 – Two Price Changes

An individual's budget line is given by:

$$p_1x_1 + p_2x_2 = m$$

Suppose the price of good 1 doubled and the price of good 2 halved. Sketch the original and new budget constraints.

Question 5 – Price and Income Changes

Suppose next year the prices of goods 1 and 2 will increase by 5% and your salary will also increase by 5%. Describe how this affects your budget line.

PREFERENCES

Question 6 – Indifference Curves

Suppose you enjoy consuming good 1 but you are not at all interested in good 2 (you are neutral about good 2). If you ever get any of good 2 you throw it straight in the trash. What would your indifference curves look like? Sketch them.

UTILITY

Question 7 – Indifference Curves and the Marginal Rate of Substitution

Answer the following questions about these utility functions:

(i) $u(x_1, x_2) = x_1 + 2x_2$.

- Sketch the indifference curves of the utility function for fixed levels of utility $k = 1$, $k = 2$ and $k = 3$.
- Calculate the marginal rate of substitution (*MRS*) between the two goods, where:

$$MRS = -\frac{\frac{\partial u(x_1, x_2)}{\partial x_1}}{\frac{\partial u(x_1, x_2)}{\partial x_2}}$$

- Can you come up with an example of two goods where this utility function would be reasonable?

(ii) $u(x_1, x_2) = \min\left\{\frac{1}{2}x_1, x_2\right\}$

- Sketch the indifference curves of the utility function for fixed levels of utility $k = 1$, $k = 2$ and $k = 3$.
- What is the marginal rate of substitution when $x_1 = 3$ and $x_2 = 1$? Interpret this number.

(iii) $u(x_1, x_2) = x_1^{\frac{1}{3}}x_2^{\frac{2}{3}}$

- Sketch the indifference curves of the utility function for fixed levels of utility $k = 1$, $k = 2$ and $k = 3$.
- Calculate the marginal rate of substitution between the two goods.