Tutorial Exercises Week 6

Question 1

Write an R function that calculates $f(x)$ in the equation below:

$$f(x) = \frac{e^x}{1 + e^x}$$

- What is $f(-2)$?
- What is $f(0)$?
- What is $f(2)$?

In each case provide your answer rounded to 4 decimal places.

* Solution

We can create the function as follows:

```
f1 <- function(x) {
  y <- exp(x) / (1 + exp(x))
  return(y)
}
```

We can evaluate the function at the 3 values with:

```
f1(c(-2, 0, 2))
```

```
[1] 0.1192029 0.5000000 0.8807971
```

We can get R to also round the output to 4 digits:

```
round(f1(c(-2, 0, 2)), digits = 4)
```

```
[1] 0.1192 0.5000 0.8808
```

Finally, it's also worth mentioning that this function has a special name. It is the *standard logistic function* and there is a built-in function in R for it called `plogis()`. We can confirm that it gives the same answers:

```r
plogis(c(-2, 0, 2))
```

```
[1] 0.1192029 0.5000000 0.8807971
```

Question 2

Plot the function

$$f(x) = x^3 - 12x^2$$

in the interval $[-6, 13]$.

At what values of x are the extreme points of this function? Both extreme points are integers (whole numbers).

Hint: Add the layers `theme_minimal() + scale_x_continuous(breaks = -6:13)` to add more line verticial breaks to help see where the extreme points are.

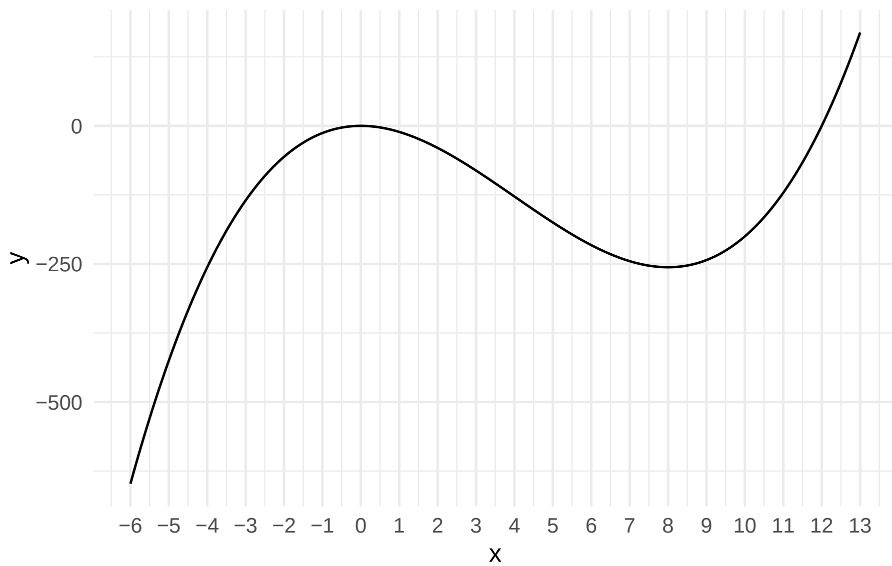
* Solution

We can create the function with:

```r
f2 <- function(x) {
  y <- x^3 - 12 * x^2
  return(y)
}
```

We then create a sequence of values `x` from -6 to 13. Going in steps of 0.1 should be good enough for the plot.

```r
library(ggplot2)
x <- seq(-6, 13, by = 0.01)
y <- f2(x)
df <- data.frame(x, y)
ggplot(df, aes(x, y)) +
  geom_line() +
  theme_minimal() +
  scale_x_continuous(breaks = -6:13)
```



We can see that there is an extreme point at $x = 0$ and at $x = 8$.

We can also do some calculus to confirm this. Taking derivatives of $f(x) = x^3 - 12x^2$:

$$f'(x) = 3x^2 - 24x$$

The extreme points occur when $f'(x) = 0$. This is when:

$$3x^2 - 24x = 0$$

Divide across both sides by 3:

$$x^2 - 8x = 0$$

We can see that both $x = 0$ and $x = 8$ solve this equation:

$$0^2 - 8 \times 0 = 0$$
$$8^2 - 8 \times 8 = 0$$

Therefore these are the extreme points.

If we take the second derivative of $f(x)$ we get:

$$f''(x) = 6x - 24$$

Using the values of the extreme points we get: $f''(0) = 6 \times 0 - 24 = -24$ and $f''(8) = 6 \times 8 - 24 = 24$.

- Because $f''(0) = -24$ is negative, this extreme point is a local maximum.
- Because $f''(8) = 24$ is positive, this extreme point is a local minimum.

This corresponds to what we see in the plot.

Question 3

Use R to find the minimum of the following function:

$$f(x) = x^2 - 3x - 12$$

- What is the minimizing value of x?
- What value does the function achieve at its minimum?

* Solution

We can create the function with:

```
f3 <- function(x) {
  y <- x^2 - 3*x - 12
  return(y)
}
```

To find the minimum and the value of the function at the minimum we use the `optimize()` function. We need to specify a wide enough interval to search over. -100 to $+100$ should be enough:

```
optimize(f3, interval = c(-100, 100), maximum = FALSE)
```

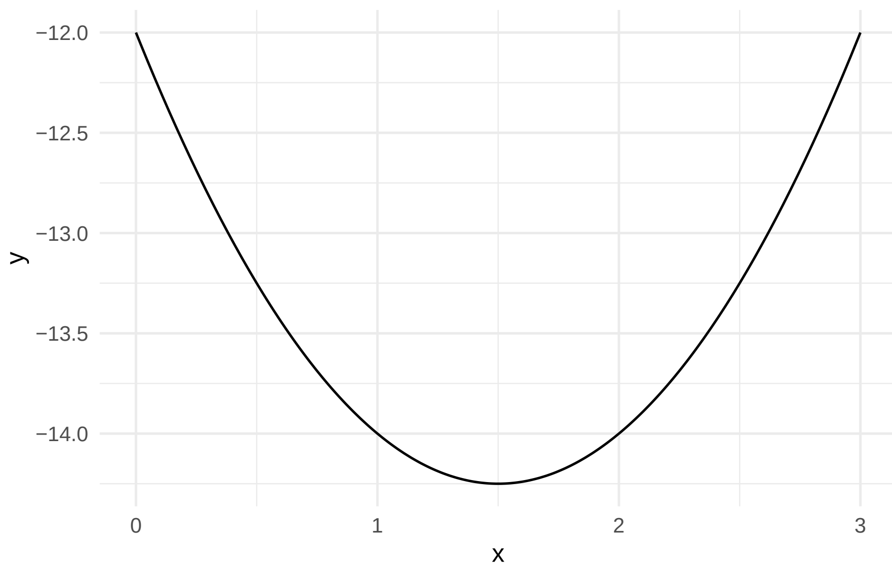
```
$minimum
[1] 1.5

$objective
[1] -14.25
```

The minimum is achieved at $x = 1.5$ and the value of $f(x)$ at $x = 1.5$ is -14.25.

We can confirm this result by plotting the function:

```
x <- seq(0, 3, by = 0.01)
y <- f3(x)
df <- data.frame(x, y)
ggplot(df, aes(x, y)) +
  geom_line() +
  theme_minimal()
```



Here we can clearly see that $x = 1.5$ minimizes the function and the function takes on a value of -14.25 at the minimum.

We can also confirm our answer by finding the minimum analytically using calculus. Taking the derivative of $f(x) = x^2 - 3x - 12$ gives:

$$f'(x) = 2x - 3$$

Extreme points occur when $f'(x) = 0$. This is when $2x - 3 = 0$, or $x = \frac{3}{2} = 1.5$. This corresponds to what we found with the `optimize()` function and the plotting approach. We can confirm that this is a minimum by taking the second derivative of $f(x)$:

$$f''(x) = 2$$

The second derivative is positive for all values of x, so the extreme point is a minimum.

Question 4

Use R to find the maximum of the following function:

$$f(x) = 100 + 2x - x^2$$

What value of x maximizes this function?

* Solution

We can create the function with:

```
f4 <- function(x) {
  y <- 100 + 2*x - x^2
  return(y)
}
```

To find value of x that maximizes this function we use the `optimize()` function with the `maximum = TRUE` option. The interval -100 to 100 should be wide enough for this problem:

```
optimize(f4, interval = c(-100, 100), maximum = TRUE)
```

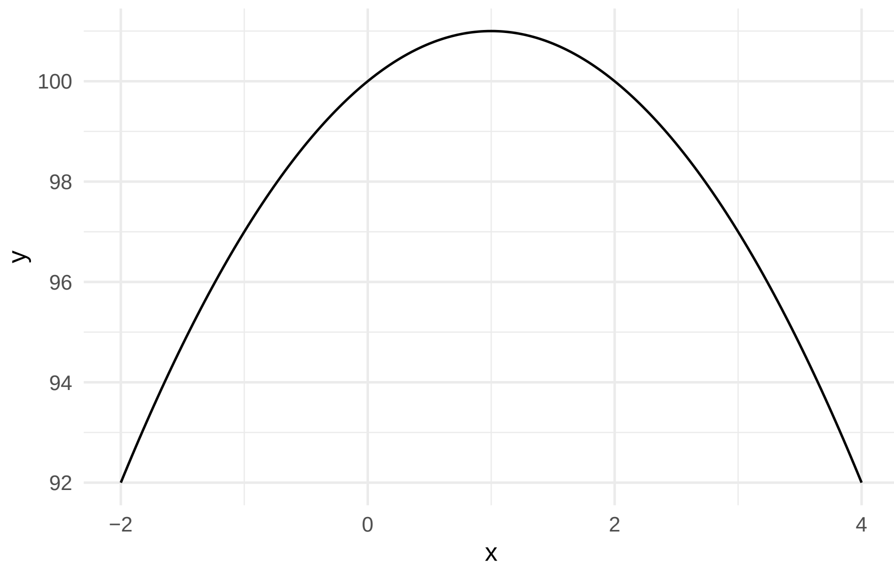
```
$maximum
[1] 1

$objective
[1] 101
```

The maximum is achieved at $x = 1$ and the value of $f(x)$ at $x = 1$ is 101.

We can confirm this result by plotting the function:

```
x <- seq(-2, 4, by = 0.01)
y <- f4(x)
df <- data.frame(x, y)
ggplot(df, aes(x, y)) + geom_line() +
  theme_minimal()
```



Here we can clearly see that $x = 1$ maximizes the function and the function takes on a value of 101 at the maximum.

We can also confirm our answer by finding the maximum analytically using calculus. Taking the derivative of $f(x) = 100 + 2x - x^2$ gives:

$$f'(x) = 2 - 2x$$

Extreme points occur when $f'(x) = 0$. This is when $2 - 2x = 0$. This occurs when $x = 1$. This corresponds to what we found with the `optimize()` function and the plotting approach. We can confirm that this is a maximum by taking the second derivative of $f(x)$:

$$f''(x) = -2$$

The second derivative is negative for all values of x, so the extreme point is a maximum.

Question 5

Create a function in R with two arguments x and z which does the following:

$$f(x, z) = \begin{cases} x + z & \text{if } x > z \\ x - z & \text{if } x = z \\ xz & \text{if } x < z \end{cases}$$

What is the value of the function at the following values:

- $x = 2$ and $z = 3$.
- $x = 2$ and $z = 2$.
- $x = 3$ and $z = 2$.

* Solution

We can create a function with more than one argument by adding the additional arguments into the `function()` function. In this case we do:

```
f5 <- function(x, z) {
  if (x > z) {
    return(x + z)
  } else if (x == z) {
    return(x - z)
  } else {
    return(x * z)
  }
}
```

We can then check the output of the function for the different values:

```
f5(x = 2, z = 3)
```

```
[1] 6
```

```
f5(x = 2, z = 2)
```

```
[1] 0
```

```
f5(x = 3, z = 2)
```

```
[1] 5
```

Question 6

Download and read in the file rotterdam-airbnb.csv.

Using the data create a factor variable called `n_beds` according to:

- `"1"` if bedrooms = 1.
- `"2"` if bedrooms = 2.
- `"3+` if bedrooms > 2.

Create a bar plot of the variable `n_beds`. Which describes the shape of the bar plot?

- Most listings have 1 bedroom. Listings with 3+ bedrooms are relatively rare.
- Most listings have 3+ bedrooms. Listings with 1 bedroom are relatively rare.
- Most listings have 2 bedrooms. Listings with 1 bedroom are relatively rare.

* Solution

We can read in the data and create the variable the following way:

```
df <- read.csv("rotterdam-airbnb.csv")
df$n_beds <- ifelse(df$bedrooms == 1, "1", ifelse(df$bedrooms == 2, "2", "3+"))
```

An equivalent way would also have been to create a blank variable and fill in the values based on the conditions:

```
df$n_beds_alt <- ""
df$n_beds_alt[df$bedrooms == 1] <- "1"
df$n_beds_alt[df$bedrooms == 2] <- "2"
df$n_beds_alt[df$bedrooms > 2] <- "3+"
```

We can see that both variables are the same by cross-tabulating them:

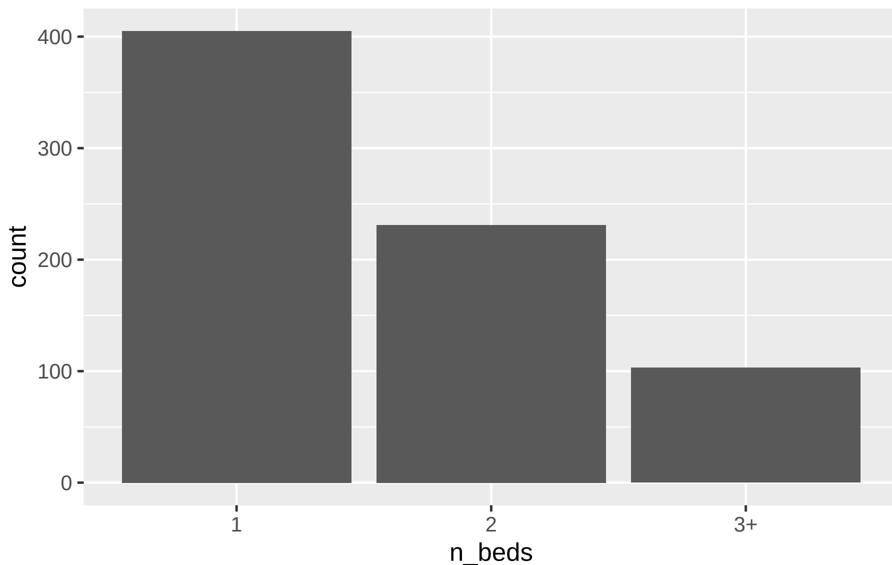
```
table(df$n_beds, df$n_beds_alt)
```

```
      1   2  3+
1   405   0   0
2     0 231   0
3+    0   0 103
```

Because all values are on the diagonal, every time `n_beds` is `"1"`, `n_beds_alt` is `"1"`, and similarly for `"2"` and `"3+"`.

But let's proceed with the original `n_beds` variable we created. We convert it to a factor variable and create the bar plot the following way:

```
df$n_beds <- factor(df$n_beds)
ggplot(df, aes(n_beds)) + geom_bar()
```



We see the bar is highest for “1” and lowest for “3+”. Thus most listings have 1 bedroom and listings with 3+ bedrooms are relatively rare.

Question 7

Using the same dataset as the previous question, create a factor variable called `affordability` which takes the following values:

- `"Cheap"` if the price is below 120
- `"Expensive"` if the price is above 250.
- `"Affordable"` otherwise (between 120-250).

Set the levels of the factor to go `"Cheap"`, then `"Affordable"`, then `"Expensive"`.

Create a bar plot of the variable `n_beds` from the previous question with colors filling the bars that represent the affordability.

Which category of `n_beds` contains the most listings labelled as `"Expensive"`?

- 1
- 2
- 3+

* Solution

We can create a character variable for the affordability as follows:

```r
df$affordability <- ifelse(df$price > 250, "Expensive",
                           ifelse(df$price < 120, "Cheap", "Affordable"))
table(df$affordability)
```

```
Affordable      Cheap  Expensive 
       431        232         76 
```

We then convert this to a factor variable specifying the order from cheap to expensive:

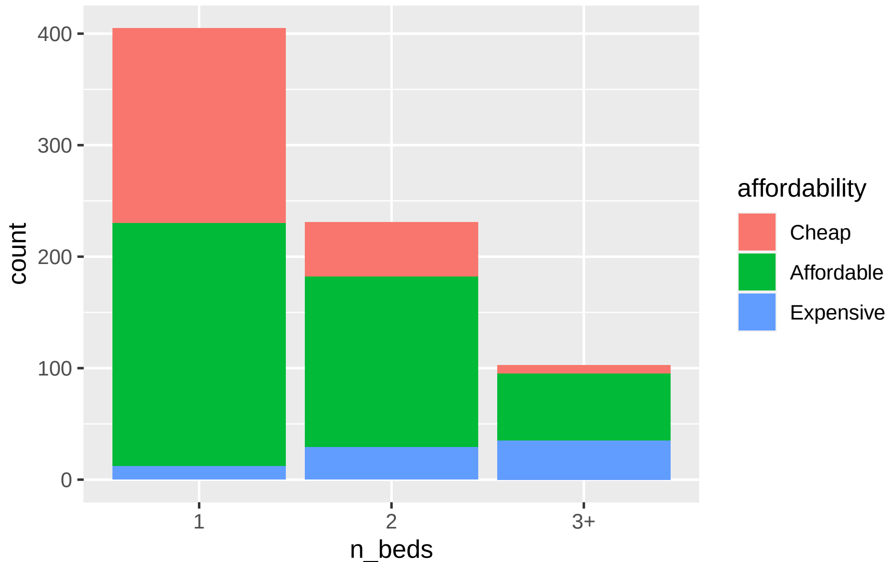
```r
df$affordability <- factor(df$affordability,
                           levels = c("Cheap", "Affordable", "Expensive"))
table(df$affordability)
```

```
     Cheap Affordable  Expensive 
       232        431         76 
```

Notice how now the `table()` function orders the output from cheap to expensive instead of alphabetically. This is because we specified the orders of the levels with the `factor()` function.

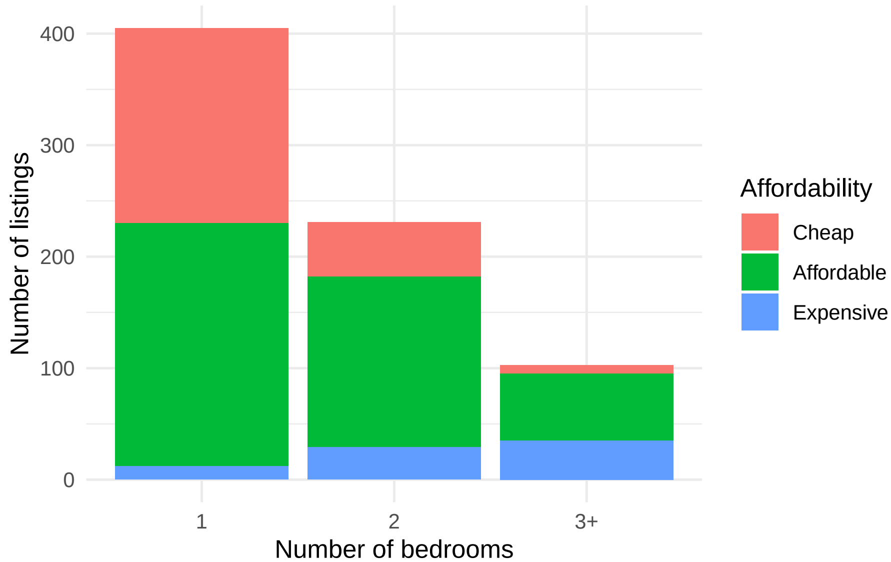
We now create the basic bar plot:

```r
ggplot(df, aes(n_beds, fill = affordability)) +
  geom_bar()
```



If we want to customize the plot a bit:

```
ggplot(df, aes(n_beds, fill = affordability)) +
  geom_bar() +
  xlab("Number of bedrooms") +
  ylab("Number of listings") +
  scale_fill_discrete(name = "Affordability") +
  theme_minimal()
```



We can see that the 1-beds have the most cheap listings, but the 3+ bedroom listings have the most expensive listings. So the answer to the question is 3+.