Elasticities

Question 1 – Cobb-Douglas Elasticities

Your utility function for goods 1 and 2 is given by:

$$u(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$$

(i) Derive the demand functions $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$.

(ii) Find the own-price elasticity of demand for good 1, ε_1.

(iii) Is good 1 an ordinary or Giffen good?

(iv) Find the the cross-price elasticity of good 1 with respect to good 2, ε_{12}.

(v) Are the substitutes, complements, or neither?

(vi) Find the income elasticity of demand for good 1, η_1.

(vii) Is the good normal or inferior?

When finding the elasticities, do not just write down the answer. Show each step in the calculation.

Question 2 – Linear demand and Marginal Revenue

The market demand curve for a good is given by:

$$D(p) = 10 - p$$

(i) What is the own-price elasticity of demand at the following price and quantity pairs:

(a) $p = 4$ and $q = 6$.

(b) $p = 5$ and $q = 5$.

(c) $p = 6$ and $q = 4$.

Comment on whether demand is elastic, inelastic or unit elastic at each of these prices.

(ii) Find the inverse demand curve, $p(q)$ for the demand curve above.

(iii) Find the revenue function, $R(q)$ for a single firm selling to this market.

(iv) Find the marginal revenue function $MR(q)$.

(v) What is the marginal revenue at each of the price and quantity pairs in (i) above?

(vi) What does your finding in (v) tell us about the relationship between elasticity and marginal revenue?

Monopoly

Question 3 – Monopoly

The (inverse) market demand for a good is given by $p(q) = 130 - q$ and there is a single producer with a cost function, $c(q) = 1600 + 10q + q^2$. Find the equilibrium price, quantity, and profits for the monopolist.

Question 4 – Monopoly vs. Perfect Competition with Linear Demand and Costs

The inverse demand curve for a good is given by:

$$p(q) = 10 - q$$

The cost function for the monopolist is:

$$c(q) = 2q$$

(i) Derive the optimal price an quantity the monopolist will choose.

(ii) Confirm that the monopolist charges a mark-up over marginal cost of $\frac{1}{1-\frac{1}{|\varepsilon|}}$. You can use your answer in Q2 (i).

(iii) Find the profit the monopolist makes.

(iv) Draw a diagram showing the following:

- The inverse demand curve.
- The marginal revenue curve.
- The marginal cost curve.
- The monopolist's price and quantity.
- The consumer surplus.
- The monopolist's profits.
- The deadweight loss.

The scaling of the diagram does not need to be exact. However, the lines/curves should have the correct shape.

(v) What is the consumer surplus?

(vi) What is the deadweight loss, if any?

Now assume the market was instead served by a large number of **perfectly competitive** firms.

(vi) What would the price and quantity be? What profit would each of the firms make?

(vii) Draw a diagram showing the following:

- The inverse demand curve.
- The marginal cost curve.
- The perfectly competitive price and quantity.
- The consumer surplus.
- The producer surplus.

(viii) What is the consumer surplus?

(ix) What is the producer surplus?

(x) What is the deadweight loss, if any?

(xi) How do each of price, quantity, profits, producer surplus, consumer surplus and deadweight loss compare between monopoly and perfect competition?

Question 5 – Natural Monopoly

The inverse demand curve for a good is given by:

$$p(q) = 10 - q$$

The cost function for the monopolist is:

$$c(q) = 9$$

(i) Derive the optimal price an quantity the monopolist will choose.

(ii) Find the profit the monopolist makes.

(iii) What is the consumer surplus?

Suppose the government introduced some regulation which forced the monopolist to charge at marginal cost.

(iv) What would the new price, quantity and profits be?

(v) What is the consumer surplus?

Suppose the government introduced some regulation which forced the monopolist to charge a price such that it makes zero profits overall.

(vi) What would the price, quantity and profits be?

(vii) What is the consumer surplus?

There are three possible policies the government can pursue, the outcomes of which you have solved for above:

- No regulation.
- Forcing the monopolist to charge at marginal cost.
- Forcing the monopolist to charge a price such that it makes zero profits.

(viii) Which policy maximizes consumer surplus?

(ix) Which policy maximizes consumer surplus *plus* monopolist profits?

Question 6 – Two-Period Monopoly with an Experience Good

Consider the following model of a two-period monopolist selling an *experience good*. The inverse market demand in the first period is

$$p(q_1) = 10 - q_1$$

In the second period the demand is:

$$p(q_2) = 10 + \frac{q_1}{2} - q_2$$

The amount the monopolist sells in the second period depends on how much the monopolist sold in the first period. The amount the monopolist sold in the first period shifts out the second period demand curve (raises the vertical intercept). The monopolist's cost function is $c(q_1, q_2) = 0$.

(i) If the monopolist was unaware he was selling an experience good and simply maximized profits in each period, what would his total profits be?

(ii) If the monopolist knows the way the demand curves depend on both quantities, write down his profit function for both periods. This should be a function of q_1 and q_2 only.

(iii) Solve for the optimal prices and quantities by maximizing the profit function you wrote down above. Remember that maximizing a function of two variables requires setting both partial derivatives equal to 0 and solving the system of equations. Does the monopolist offer a low introductory price?

(iv) Compare the profits from parts (i) and (iii). Is the monopolist better off in (i) or (iii)?