# Choice and Demand

## Question 1 – Neutral Goods

Revisiting the preferences from Question 6 on the previous problem set, suppose that you enjoy good 1 but you are neutral about good 2 (your indifference curves are vertical lines).

(i) Find the demand functions $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$.

(ii) Write down a utility function $u(x_1, x_2)$ that would be consistent with these preferences.

## Question 2 – Cobb-Douglas Demand

Suppose you have the following utility function for goods 1 and 2:

$$u(x_1, x_2) = 2x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$$

(i) Find the demand functions $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$.

(ii) The price of good 1 is \$1 and the price of good two is \$3. You have \$60 to spend. How much will you consume of goods 1 and 2?

(iii) How much of goods 1 and 2 will you buy if you had \$120 and the prices stayed the same? Are the goods normal or inferior?

(iv) If the price of good 2 doubled (to \$6), what would happen to your demand for good 1?

(v) Which of the following statements is true and why?

- Good 1 is a substitute to good 2.
- Good 1 is a complement to good 2.
- Good 1 is neither a substitute nor a complement to good 2.

## Question 3 – Income and Substitution Effects

Suppose your utility function is $u(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$. It can be shown that the resulting demand functions are $x_1(p_1, p_2, m) = \frac{2m}{3p_1}$ and $x_2(p_1, p_2, m) = \frac{m}{3p_2}$ (you should check this). Throughout this question, you have income of $m = 30$.

(i) If $p_1 = 1$ and $p_2 = 1$, how much of goods 1 and 2 would you buy? Sketch the budget line and optimal bundle on a graph.

(ii) If the price of good 1 increases to $p_1 = 2$ and the price of good 2 remains at $p_2$, how much of goods 1 and 2 would you buy? Sketch the new budget line and optimal bundle on your graph.

(iii) Suppose that after the price of good 1 increased, the government gave you just enough money so that you could afford the original bundle, i.e. the bundle in part(i). How much money would the government need to give you?

(iv) If the government gave you this reimbursement, how much of goods 1 and 2 would you buy? Is this different from part (i)? Sketch your new budget line after the government handout and the optimal bundle.

(v) What is the income effect and substitution effect of the price increase of good 1 on the demand for good 1?

## Intertemporal Choice

### Question 4 – Saving for Retirement

There are two time periods. In period 1 you work and earn income $m_1$. In period 2 you are retired and earn no income ($m_2 = 0$). The interest rate is $r$ and there is no inflation. Your consumption in time periods 1 and 2 are $c_1$ and $c_2$ respectively.

The lifetime budget constraint is therefore:

$$c_1 + \frac{c_2}{1+r} = m_1$$

Your lifetime utility function is:

$$u(c_1, c_2) = \frac{1}{2}c_1^2 + \frac{\beta}{2}c_2^2$$

where $\beta$ is a constant.

(i) Sketch the budget constraint on a graph. Label the axis intercepts and the endowment.

(ii) The marginal rate of substitution of consumption between the two periods is:

$$MRS = -\frac{\frac{\partial u(c_1,c_2)}{\partial c_1}}{\frac{\partial u(c_1,c_2)}{\partial c_2}}$$

Calculate the *MRS* for the lifetime utility function $u(c_1, c_2)$ provided above.

(iii) Now we want to find exactly how much you will want to save in the first period. In what follows, assume that $\beta = \frac{1}{1+r}$ (this will make the algebra easier).

At the optimum you will set the *MRS* equal to the slope of the budget constraint, which is $-(1+r)$. Therefore we have two equations to find $c_1$ and $c_2$:

$$\frac{\frac{\partial u(c_1,c_2)}{\partial c_1}}{\frac{\partial u(c_1,c_2)}{\partial c_2}} = 1 + r \tag{1}$$

$$c_1 + \frac{c_2}{1+r} = m_1 \tag{2}$$

Using these two equations, solve for $c_1$ and $c_2$ as a function of $m_1$ and $r$.

(iv) If the interest rate increases, will you consume more or less in the first period? To answer this you can assume that $m_1 = 1$ and $r$ ranges from 0 to 1.

## Uncertainty

### Question 5 – Insurance

You have \$200,000 worth of valuable items in your home. There is a 5% chance that you will be burgled. Fortunately, you keep \$100,000 worth of your valuables in a hidden safe that the burgulars won't find. The local insurance company will insure your valuables for \$7,000. That is, in the event of a burgulary the insurance company will give you \$100,000 (the amount the burgulars stole).

Suppose your utility for wealth is $U(W) = \sqrt{W}$. So if you have \$200,000 you utility is $\sqrt{200,000}$.

(i) What is your expected utility if you don't purchase insurance?

(ii) What is your expected utility if you do purhcase insurance?

(iii) Would you purchase insurance?

(iv) How much would the insurance policy be if it were actuarially fair?

## Technology

### Question 6 – The Technical Rate of Substitutition (TRS)

Find the techinical rate of substitution for the following production function (show all your work, including the partial derivatives to get the marginal products):

$$f(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$$

Interpret what the *TRS* means when $x_1 = 2$ and $x_2 = 4$.

### Question 7 – Returns to scale

Show whether the following production functions exhibit constant returns to scale, increasing returns to scale or decreasing returns to scale:

(i) $f(x_1, x_2) = x_1 x_2$.

(ii) $f(x_1, x_2) = x_1 + x_2$.

(iii) $f(x_1, x_2) = \min\{x_1, x_2\}$.

## Profit Maximization

### Question 8 – Short-run profit maximization

Suppose the production function is $y = f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$, the price of output is $p = 2$ and the prices of each input/factor are $w_1 = 1$ and $w_2 = 1$. The firm is competitive and therefore has no control over input and output prices. Factor 2 is fixed in the short run at $x_2 = 1$. Find the amount of factor 1 that will maximimize the firm's profits. What profit does the firm make?