Short Questions

Question 1 – Preferences (5 Points)

The following are common assumptions about preferences:

- Completeness
- Transitivity
- Monotonicity
- Convexity

Consider the following scenario:

- I ask you which of the bundles $(x_1, x_2) = (2, 2)$ and $(x_1, x_2) = (1, 5)$ you prefer and you tell me you strictly prefer $(x_1, x_2) = (2, 2)$.
- I ask you which of the bundles $(x_1, x_2) = (1, 5)$ and $(x_1, x_2) = (5, 1)$ you prefer and you tell me that you strictly prefer $(x_1, x_2) = (1, 5)$.
- I ask you which of the bundles $(x_1, x_2) = (5, 1)$ and $(x_1, x_2) = (2, 2)$ you prefer and you tell me that you strictly prefer $(x_1, x_2) = (5, 1)$.

Which one of the common assumptions listed above do your choices violate and why? From your choices we can gather the following:

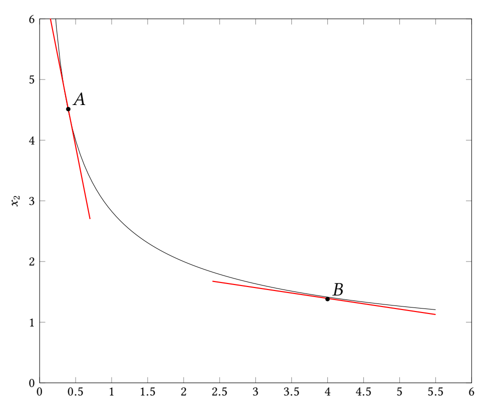
- $(2, 2) \succ (1, 5)$
- $(1, 5) \succ (5, 1)$
- $(5, 1) \succ (2, 2)$

Transitivity in short says that if $A \succ B$ and $B \succ C$, then it must be the case that $A \succ C$. Here it means that if you prefer $(2, 2)$ to $(1, 5)$ and $(1, 5)$ to $(5, 1)$, then it must be the case that you prefer $(2, 2)$ to $(5, 1)$. However in the 3rd choice scenario, you chose $(5, 1)$ over $(2, 2)$, violating transitivity.

This doesn't violate completeness because you have a preference. This doesn't violate monotonicity because in no case did a bundle have at least as much of both goods with strictly more of one good. It doesn't violate convexity because one bundle cannot be written as a convex combination (a weighted average) of the other bundles.

Question 2 – Technology (5 Points)

The graph below shows an isoquant for a production function $f(x_1, x_2)$.



What do the slopes of the tangents at points A and B represent? How do we interpret it?

The slopes of the tangent of an isoquant represents the technical rate of substitution (TRS). If we reduce x_1 a marginal amount, it measures how much extra x_2 we need to keep output constant. You could also have said that the move from A to B represents the diminishing rate of technical substitution.

For a longer answer (not required to get full points): The magnitude of the slope at A is larger than at B for the following reason. At A we are currently using a lot of x_2 and not very much x_1. Since both x_1 and x_2 are needed for production (this is a convex production technology), if we reduce x_1 even further we need a large amount of x_2 to compensate us in order to be able to produce the same amount of output. At point B we are using a lot of x_1 and not so much x_2. If we reduce x_1 a small amount we do not need to be compensated with very much x_2 in order to be able to produce the same amount of output.

Long Questions

Question 3 – Choice (15 Points)

Your utility function for goods 1 and 2 is:

$$u(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$$

The prices of goods 1 and 2 are $p_1 = 2$ and $p_2 = 1$ respectively. You have $m = 30$ income to spend.

(i) **[5 Points]** Find the marginal utility for goods 1 and 2, MU_1 and MU_2.

$$MU_1 = \frac{\partial u(x_1, x_2)}{\partial x_1} = \frac{2}{3} x_1^{\frac{2}{3}-1} x_2^{\frac{1}{3}} = \frac{2}{3} x_1^{-\frac{1}{3}} x_2^{\frac{1}{3}}$$

$$MU_2 = \frac{\partial u(x_1, x_2)}{\partial x_2} = \frac{1}{3} x_1^{\frac{2}{3}} x_2^{\frac{1}{3}-1} = \frac{1}{3} x_1^{\frac{2}{3}} x_2^{-\frac{2}{3}}$$

(ii) **[5 Points]** Find the marginal rate of substitution, MRS.

$$MRS = -\frac{MU_1}{MU_2} = -\frac{\frac{2}{3} x_1^{\frac{2}{3}-1} x_2^{\frac{1}{3}}}{\frac{1}{3} x_1^{\frac{2}{3}} x_2^{-\frac{2}{3}}} = -2\frac{x_1^{-1}}{x_2^{-1}} = -\frac{2x_2}{x_1}$$

(iii) **[5 Points]** How much of goods 1 and 2 will you demand? Do not just write down the final answer. Derive the demand from the budget constraint and the consumer's optimality/tangency condition. The budget constraint is:

$$p_1 x_1 + p_2 x_2 = m$$

Using the values for p_1, p_2 and m:

$$2x_1 + x_2 = 30$$

The optimality condition is that we need to set $|MRS| = \frac{p_1}{p_2}$:

$$\frac{2x_2}{x_1} = 2$$

This is just $x_1 = x_2$. You will want to consume x_1 and x_2 in equal amounts. Using this in the budget constraint:

$$2x_1 + x_2 = 30 \iff 2x_1 + x_1 = 30 \iff 3x_1 = 30 \iff x_1 = 10$$

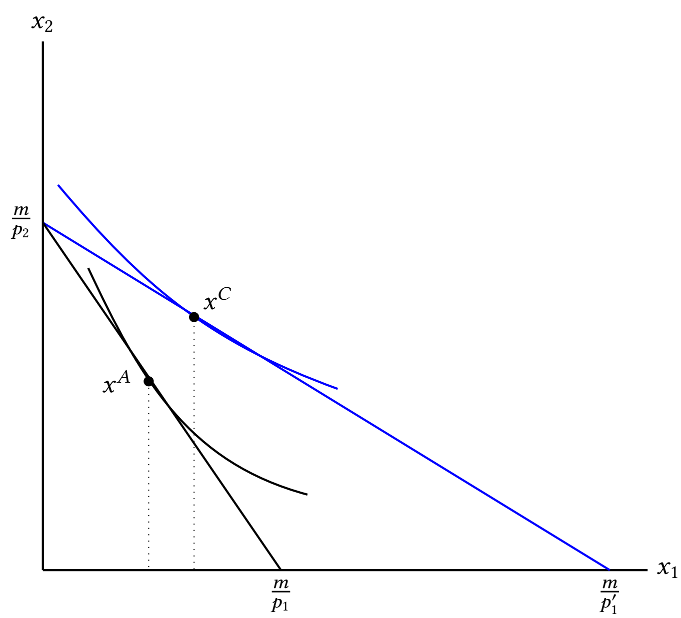
And since $x_2 = x_1$, $x_2 = 10$.

Question 4 – Income and Substitution Effects (15 Points)

There are two goods with prices p_1 and p_2. You have income m. According to your preferences for goods 1 and 2, good 1 is inferior (but not Giffen) and good 2 is a normal good. Your optimal bundle is a combination (x_1, x_2). If the price of good 1 falls from p_1 to p_1', you end up consuming *more* of both goods 1 and 2, i.e. your optimal bundle is $\left(x_1', x_2'\right)$ where $x_1' > x_1$ and $x_2' > x_2$.

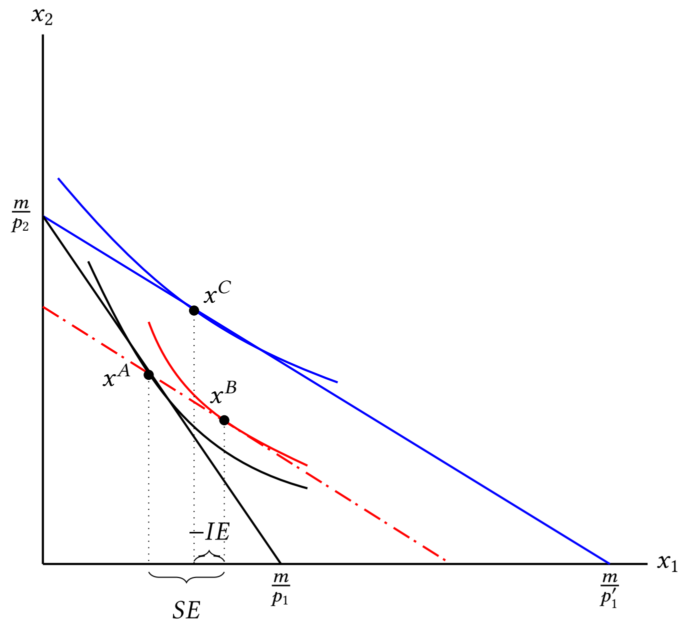
(i) **[5 Points]** Draw a graph showing the following:

- The budget constraints before the price of good 1 falls and after the price of good 1 falls. Label the vertical and horizontal intercepts.
- The optimal bundles before and after the price change, as well as the indifference curves associated with those optimal bundles.



(ii) **[7 Points]** Draw a separate graph for this question. You will redraw your graph from part (i) but you will add the following:

- Show the income and substitution effect decomposition, i.e. show the budget line that brings you back to your original purchasing power but with the new relative prices, the optimal bundle that you would choose on this budget line, and the indifference curve associated with that bundle. Identify where the income effect is and where the substitution effect is. **Remember, good 1 is an *inferior* good**.



(iii) **[3 Points]** Compare the relative magnitudes of the income and substitution effect.

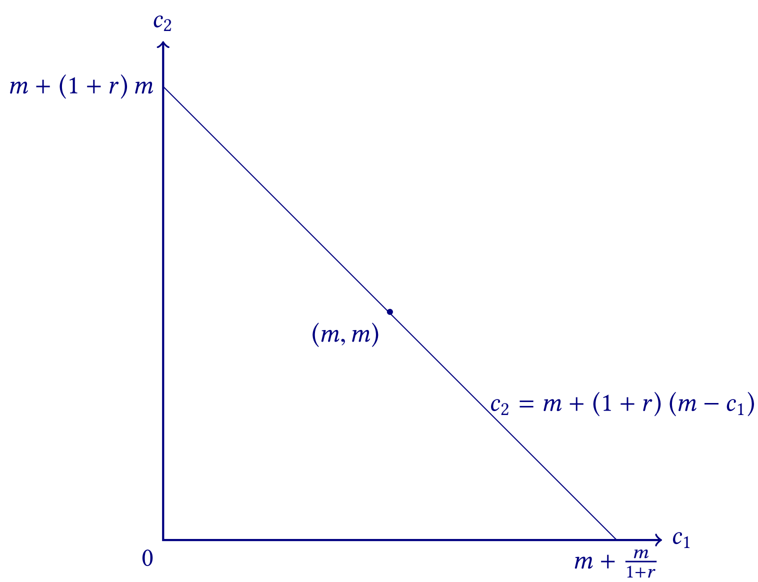
The substitution effect is positive and the income effect is negative. However, the magnitude of the substitution effect is larger than the income effect.

Question 5 – Intertemporal Choice (15 Points)

*In this question, please draw **separate** graphs for each part (i), (ii) and (iii).*

There are two periods. You receive income m in period 1 and m in period 2 (you receive the same amount of money in both periods, so in total you get $2m$ in your lifetime). You are able to borrow and lend at an interest rate r. There is no inflation.

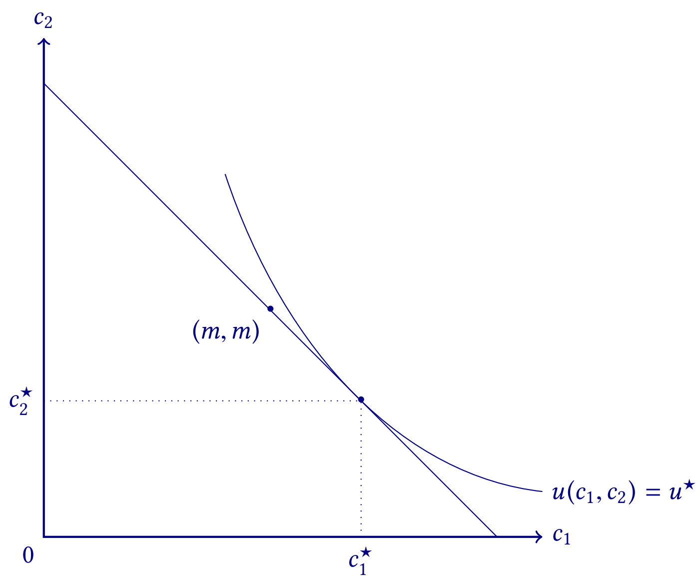
(i) **[5 Points]** Draw the budget constraint. Label the axis intercepts and the endowment.



Your preferences, $u(c_1, c_2)$, endowment, (m_1, m_2), and the interest rate, r, lead to you optimally choose a consumption path (c_1, c_2) where you **borrow** in the first period. That is, $c_1 > m$.

(ii) **[5 Points]** Draw your optimal choice on a graph. Show the budget constraint, endowment, the optimal choice, and the indifference curve associated with the optimal choice.

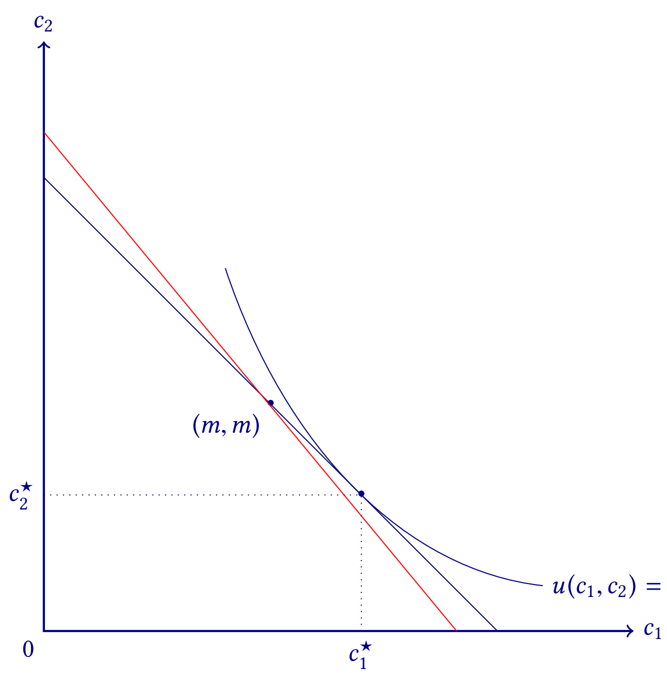
The optimal choice is where the indifference curve is tangent to the budget constraint. You borow in the first period so the optimal choice is *south-east* of the endowment:



Now the interest rate r, increases.

(iii) **[5 Points]** Show the change in the budget constraint on a graph. Can you still aford your original bundle?

With an interest rate increase, the budget line pivots around the endownment.



You can no longer afford your optimal bundle from before.

Question 6 – Uncertainty (15 Points)

You have \$20,000 in your bank account. Your car is worth \$10,000 (so altogether your wealth is \$30,000). The probablity that your car will be stolen in a given year is 10%. An insurance company offers to insure your car against theft for $1,050 per year. Your utility for wealth is $U(W) = \sqrt{W}$.

(i) **[2 Points]** Are you risk averse, risk loving or risk neutral? Explain why.

Your utility function is concave so you are risk averse.

(ii) **[4 Points]** What is your expected utility from *not* purchasing insurance?

If you do not purchase insurance, in the good state you keep your money in your bank account and your car. So you get $30,000. In the bad state, your car is stolen and you only keep the money in your bank account. So you get $20,000. Therefore your expected utility is:

$$0.9\sqrt{30000} + 0.1\sqrt{20000} = 170.0267$$

(iii) **[4 Points]** What is your expected utility from purchasing insurance?

If you purchase insurance, you get $30,000-$1,100=$18,900 in both states. You get $28,900 for certain. Your expected utility is then:

$$\sqrt{28950} = 170.147$$

(iv) **[2 Points]** Will you purchase insurance?

Your expected utility from purchasing insurance is higher than if you didn't so you will purchase insurance.

(v) **[3 Points]** How much would the actuarially fair insurance policy cost?

The actuarially fair insurance costs what the expected value of the damage is. Here it is $0.1 \times \$10,000 = \$1,000$.

Question 7 – Cost curves, Firm Supply and Industry Supply (15 Points)

There are 100 firms in a particular perfectly competitive industry. Each firm has the following cost function:

$$c(y) = 2y^2 + 4$$

The equilibrium price of output is p.

(i) **[2 Points]** What is the variable cost function $c_v(y)$?

The variable cost function is $c_v(y) = 2y^2$.

(ii) **[2 Points]** What is the firm's fixed cost?

The fixed cost is 4, the constant term in the cost function.

(iii) **[3 Points]** What is the average cost function, $AC(y)$?

$$AC(y) = \frac{c(y)}{y} = \frac{2y^2 + 4}{y} = 2y + \frac{4}{y}$$

(iv) **[3 Points]** What is the marginal cost function, $MC(y)$?

$$MC(y) = \frac{dc(y)}{dy} = 4y$$

(v) **[3 Points]** What is firm i's supply function, $S_i(p)$ (the supply function for one individual firm)?

The firm will set $p = MC(y)$. So:

$$p = 4y \iff y = \frac{4}{p} \iff S_i(p) = \frac{p}{4}$$

(vi) **[2 Points]** What is the industry supply function, $S(p)$?

The industry supply function adds up all the individual supply functions:

$$S(p) = S_1(p) + S_2(p) + \cdots + S_{100}(p) = \frac{p}{4} + \frac{p}{4} + \cdots + \frac{p}{4} = 100 \times \frac{p}{4} = \frac{25}{p}$$

Question 8 – Equilibrium and Taxes (15 Points)

The demand and supply functions for a particular good in the market are given by:

$$D(p) = 24 - 4p$$
$$S(p) = 4 + 6p$$

(i) **[6 Points]** Find the equilibrium price and quantity.

To find the equilibrium price, $p^\star$, we set $D(p^\star) = S(p^\star)$:

$$24 - 4p^\star = 4 + 6p^\star \iff 24 - 4 = 4p^\star + 6p^\star \iff 20 = 10p^\star \iff p^\star = \frac{20}{10} \iff p^\star = 2$$

The quantity can be found by using $p^\star$ in either $S(p)$ or $D(p)$. Using demand:

$$q^\star = 24 - 4 \times 2 = 16$$

Using supply:

$$q^\star = 4 + 6 \times 2 = 16$$

Now the government imposes a per-unit tax (a quantity tax) of 1 on the good.

(ii) **[6 Points]** Find the price the buyers pay, the price sellers receive and the new equilibrium quantity.

Now $P_D = P_S + 1$. We can replace p in the demand function with $P_S + 1$ and then set demand equal to supply:

$$24 - 4(P_S + 1) = 4 + 6P_S$$
$$24 - 4P_S - 4 = 4 + 6P_S$$
$$16 = 10P_S$$
$$P_S = 1.6$$

Using $P_D = P_S + 1$ we can find the price buyers pay:

$$P_D = 1.6 + 1 = 2.6$$

The price increased for buyers by 60 cents and the price decreased for sellers by 40 cents.

The new equilibrium quantity can be found using either the inverse demand or supply function. Using demand:

$$q^\star = 24 - 4P_D = 2t - 4 \times 2.6 = 24 - 10.4 = 13.6$$

Using supply:

$$q^\star = 4 + 6P_S = 4 + 6 \times 1.6 = 4 + 9.6 = 13.6$$

The tax decreased the equilibrium quantity by 2.4 units.

(iii) **[3 Points]** What portion of the tax do consumers pay and what portion of the tax do sellers pay?

Since the price increased by $\frac{3}{5}$ of the tax for consumers, they pay 60% of the tax. The price decreased by $\frac{2}{5}$ of the tax for sellers so they pay 40% of the tax.