Question 1 – Preferences (10 Points)

The following are common assumptions about preferences:

- Completeness
- Transitivity
- Monotonicity
- Convexity

Consider the following scenario:

- I ask you which of the bundles $(x_1, x_2) = (2, 8)$ and $(x_1, x_2) = (8, 2)$ you prefer and you tell me you strictly prefer $(x_1, x_2) = (2, 8)$.
- I ask you which of the bundles $(x_1, x_2) = (8, 2)$ and $(x_1, x_2) = (5, 5)$ you prefer and you tell me that you strictly prefer $(x_1, x_2) = (8, 2)$.
- I ask you which of the bundles $(x_1, x_2) = (2, 8)$ and $(x_1, x_2) = (5, 5)$ you prefer and you tell me that you strictly prefer $(x_1, x_2) = (5, 5)$.

Which one of the common assumptions listed above do your choices violate and why?

From your choices we can gather the following:

- $(2, 8) \succ (8, 2)$
- $(8, 2) \succ (5, 5)$
- $(5, 5) \succ (2, 8)$

Transitivity in short says that if $A \succ B$ and $B \succ C$, then it must be the case that $A \succ C$. Here it means that if you prefer $(2, 8)$ to $(8, 2)$ and $(8, 2)$ to $(5, 5)$, then it must be the case that you prefer $(2, 8)$ to $(5, 5)$. However in the 3rd choice scenario, you chose $(5, 5)$ over $(2, 8)$, violating transitivity.

That's all you needed to say to get full points for this question. But for further explanation we can see why these choices don't violate any of the other assumptions.

- It doesn't violate completeness because you were always able to make a comparison between the two alternatives.
- It doesn't violate monotonicity because we never had a case where one of the two alternatives had at least as much of both goods and strictly more of at least one. Two examples of choices that *would* violate monotonicity are:
  - $(2, 2) \succ (4, 2)$
  - $(2, 2) \succ (4, 4)$

However, in every pair of bundles in the quetsion one bundle had more of one good and less of the other, so monotonicity could never have been violated.

- Recall how we defined convexity in class. We said that if you are indifferent between two extreme bundles, then the average bundle is at least as good. So:

$$\text{If } A \sim B, \text{ then } \frac{1}{2}A + \frac{1}{2}B \succeq A$$

So for example if $A = (2, 8)$ and $B = (8, 2)$, then the average bundle, i.e. $(5, 5)$, must be at least as good. However, here we don't have that you are indifferent between $(8, 2)$ and $(2, 8)$, but rather you strictly prefer one to the other. Therefore we can't say anything about violating convexity in this case.
A few of you said that since $(8, 2) \succ (5, 5)$, that that violates convexity. This on its own doesn't violate convexity. It might be the case that good 1 gives much more utility than good 2, and if you took the average of the two you would be worse off. You need to be indifferent between the two extreme bundles before you can say that you should prefer the average bundle.
In the practice midterm there was a violation of convexity because in the first choice you said you were indifferent between the two extreme bundles, $(2, 8) \sim (8, 2)$ but then in the second choice you said $(2, 8) \succ (5, 5)$.

Question 2 – Choice (20 Points)

Your utility function for goods 1 and 2 is:

$$u(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$$

The prices of goods 1 and 2 are $p_1 = 2$ and $p_2 = 1$ respectively. You have $m = 30$ income to spend.

(i) **[6 Points]** Find the marginal utility for goods 1 and 2, MU_1 and MU_2.

$$MU_1 = \frac{\partial u(x_1, x_2)}{\partial x_1} = \frac{2}{3} x_1^{\frac{2}{3}-1} x_2^{\frac{1}{3}} = \frac{2}{3} x_1^{-\frac{1}{3}} x_2^{\frac{1}{3}}$$

$$MU_2 = \frac{\partial u(x_1, x_2)}{\partial x_2} = \frac{1}{3} x_1^{\frac{2}{3}} x_2^{\frac{1}{3}-1} = \frac{1}{3} x_1^{\frac{2}{3}} x_2^{-\frac{2}{3}}$$

(ii) **[6 Points]** Find the marginal rate of substitution, MRS.

$$MRS = -\frac{MU_1}{MU_2} = -\frac{\frac{2}{3} x_1^{\frac{2}{3}-1} x_2^{\frac{1}{3}}}{\frac{1}{3} x_1^{\frac{2}{3}} x_2^{-\frac{2}{3}}} = -2\frac{x_1^{-1}}{x_2^{-1}} = -\frac{2x_2}{x_1}$$

(iii) **[8 Points]** How much of goods 1 and 2 will you demand? Do not just write down the final answer. Derive the demand from the budget constraint and the consumer's optimality/tangency condition.

The budget constraint is:

$$p_1 x_1 + p_2 x_2 = m$$

Using the values for p_1, p_2 and m:

$$2x_1 + x_2 = 30$$

The optimality condition is that we need to set $|MRS| = \frac{p_1}{p_2}$:

$$\frac{2x_2}{x_1} = 2$$

This is just $x_1 = x_2$. You will want to consume x_1 and x_2 in equal amounts. Using this in the budget constraint:

$$2x_1 + x_2 = 30 \iff 2x_1 + x_1 = 30 \iff 3x_1 = 30 \iff x_1 = 10$$

And since $x_2 = x_1$, $x_2 = 10$.

Question 3 – Demand (15 Points)

Your demand functions for goods 1 and 2 are:

$$x_1(p_1, p_2, m) = \frac{2m - 100}{p_1}$$
$$x_2(p_1, p_2, m) = \frac{100 - m}{p_2}$$

Assume that $50 \leq m \leq 100$ (this is not required to answer the question, but rather it is a a technical condition so that your demand for both goods is never negative).

(i) **[3 Points]** Is good 1 an ordinary or a Giffen good?

If p_1 increases, then the demand for good 1 decreases. Therefore good 1 is an ordinary good.

(ii) **[3 Points]** Is good 2 an ordinary or a Giffen good?

If p_2 increases, then the demand for good 2 decreases. Therefore good 2 is an ordinary good.

(iii) **[3 Points]** Is good 1 a normal good or an inferior good?

If m increases, then the demand for good 1 increases. Therefore good 1 is a normal good.

(iv) **[3 Points]** Is good 2 a normal good or an inferior good?

If m increases, then the demand for good 2 decreases. Therefore good 2 is a normal good.

(v) **[3 Points]** Is good 1 a substitute, a complement, or neither, for good 2?

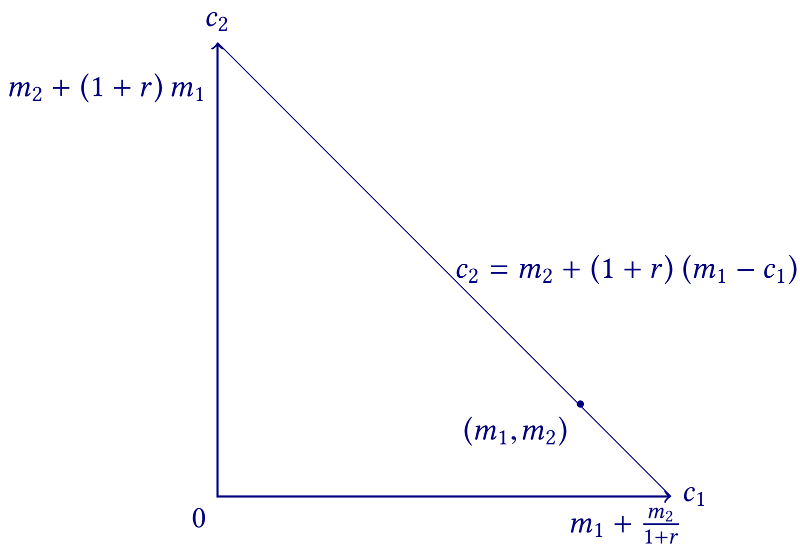
If p_2 changes, then the demand for good 1 doesn't change. Similarly, if p_1 changes, the demand for good 2 doesn't change. Therefore the two goods are neither substitutes nor complements.

Question 4 – Intertemporal Choice (15 Points)

*In this question, please draw **separate** graphs for each part (i), (ii) and (iii).*

There are two periods. You receive income m_1 in period 1 and m_2 in period 2. You are able to borrow and lend at an interest rate r. There is no inflation.

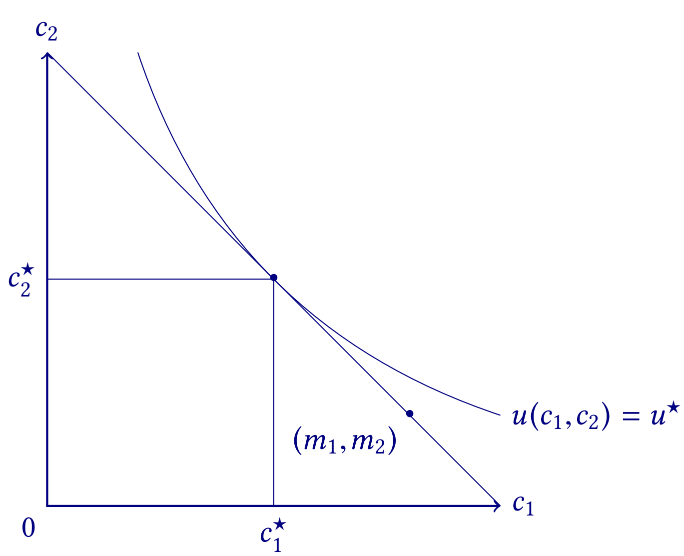
(i) **[5 Points]** Draw the budget constraint. Label the axis intercepts and the endowment.



Your preferences, $u(c_1, c_2)$, endowment, (m_1, m_2), and the interest rate, r, lead to you optimally choose a consumption path (c_1, c_2) where you **save** in the first period. That is, $c_1 < m_1$.

(ii) **[5 Points]** Draw your optimal choice on a graph. Show the budget constraint, endowment, the optimal choice, and the indifference curve associated with the optimal choice.

The optimal choice is where the indifference curve is tangent to the budget constraint. You save in the first period so the optimal choice is *north-west* of the endowment:

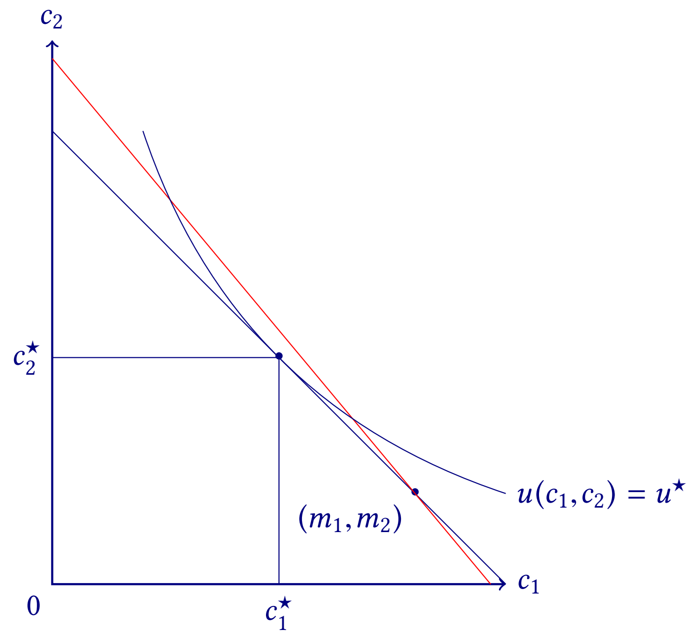


Now the interest rate r, increases.

(iii) **[5 Points]** Show the change in the budget constraint on a graph. Will you be able to choose a consumption path that gives you higher utility?

An increase in the interest rate pivots the budget constraint around the endowment. If you only borrowed, you can consume less. If you only saved, you can consume more. If you stayed at your endowment, the interest rate doesn't affect you.

This is shown in red in the diagram below. The change in the budget constraint means that consumption paths that are better than the previous consumption path are now available. We can move to a new consumption path on the new budget constraint.



Question 5 – Uncertainty (15 Points)

You have \$20,000 in your bank account. Your car is worth \$10,000 (so altogeher your wealth is \$30,000). The probablity that your car will be stolen in a given year is 10%. An insurance company offers to insure your car against theft for \$1,050 per year. Your utility for wealth is $U(W) = \sqrt{W}$.

(i) **[2 Points]** Are you risk averse, risk loving or risk neutral?

You utility function is concave so you are risk averse.

(ii) **[4 Points]** What is your expected utility from *not* purchasing insurance?

If you do not purchase insurance, in the good state you keep your money in your bank account and your car. So you get \$30,000. In the bad state, your car is stolen and you only keep the money in your bank account. So you get \$20,000. Therefore your expected utility is:

$$0.9\sqrt{30000} + 0.1\sqrt{20000} = 170.0267$$

(iii) **[4 Points]** What is your expected utility from purchasing insurance?

If you purchase insurance, you get \$30,000-\$1,100=\$18,900 in both states. You get \$28,900 for certain. Your expected utility is then:

$$\sqrt{28950} = 170.147$$

(iv) **[2 Points]** Will you purchase insurance?

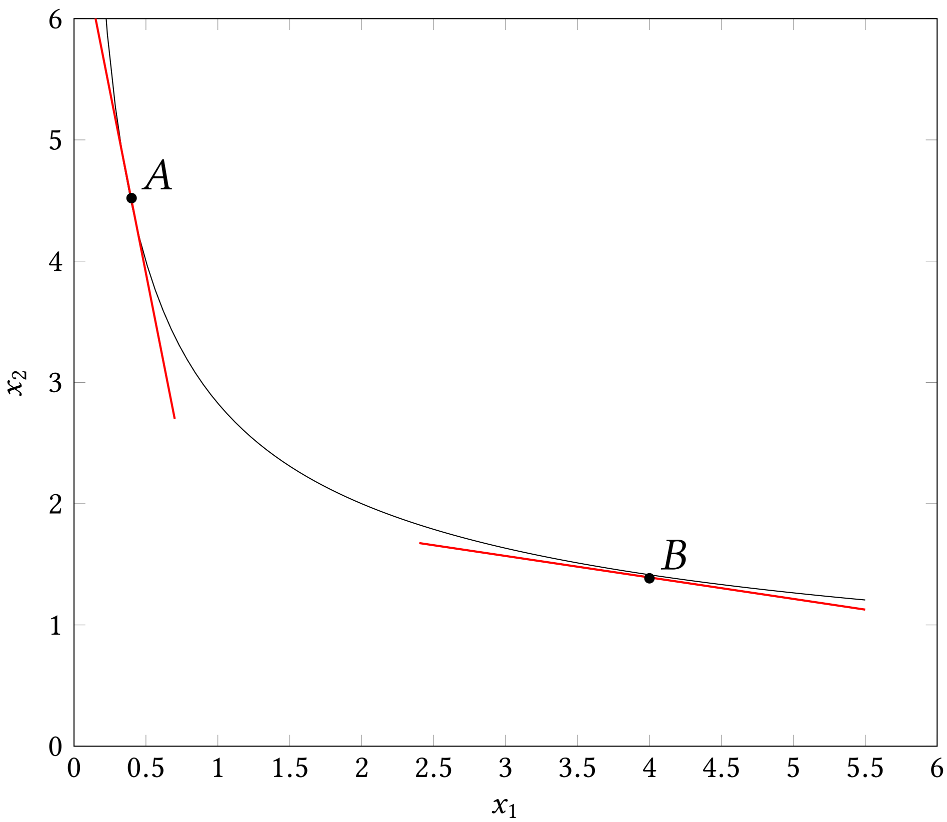
Your expected utility from purchasing insurance is higher than if you didn't so you will purchase insurance.

(v) **[3 Points]** How much would the actuarially fair insurance policy cost?

The actuarially fair insurance costs what the expected value of the damage is. Here it is $0.1 \times \$10,000 = \$1,000$.

Question 6 – Technology (10 Points)

The graph below shows an isoquant for a production function $f(x_1, x_2)$.



What do the slopes of the tangents at points A and B represent? How do we interpet it?

The slopes of the tangent of an isoquant represents the technical rate of substitution (TRS). If we reduce x_1 a marginal amount, it measures how much extra x_2 we need to keep output constant.

The magnitude of the slope at A is larger than at B for the following reason. At A we are currently using a lot of x_2 and not very much x_1. Since both x_1 and x_2 are needed for production (this is a convex production technology), if we reduce x_1 even further we need a large amount of x_2 to compensate us in order to be able to produce the same amount of output. At point B we are using a lot of x_1 and not so much x_2. If we reduce x_1 a small amount we do not need to be compensated with very much x_2 in order to be able to produce the same amount of output.

Question 7 – Cost curves, Firm Supply and Industry Supply (20 Points)

There are 100 firms in a particular perfectly competitive industry. Each firm has the following cost function:

$$c(y) = 2y^2 + 4$$

The equilibrium price of output is p.

(i) **[3 Points]** What is the variable cost function $c_v(y)$?

The variable cost function is $c_v(y) = 2y^2$.

(ii) **[3 Points]** What is the firm's fixed cost?

The fixed cost is 4.

(iii) **[3 Points]** What is the average cost function, $AC(y)$?

$$AC(y) = \frac{c(y)}{y} = \frac{2y^2 + 4}{y} = 2y + \frac{4}{y}$$

(iv) **[3 Points]** What is the marginal cost function, $MC(y)$?

$$MC(y) = \frac{dc(y)}{dy} = 4y$$

(v) **[4 Points]** What is firm i's supply function, $S_i(p)$ (the supply function for one individual firm)?

The firm will set $p = MC(y)$. So:

$$p = 4y \iff y = \frac{4}{p} \iff S_i(p) = \frac{p}{4}$$

(vi) **[4 Points]** What is the industry supply function, $S(p)$?

The industry supply function adds up all the individual supply functions:

$$S(p) = S_1(p) + S_2(p) + \cdots + S_{100}(p) = \frac{p}{4} + \frac{p}{4} + \cdots + \frac{p}{4} = 100 \times \frac{p}{4} = \frac{25}{p}$$

Question 8 – Equilibrium and Taxes (15 Points)

The demand and supply functions for a particular good in the market are given by:

$$D(p) = 24 - 4p$$
$$S(p) = 4 + 6p$$

(i) **[5 Points]** Find the equilibrium price and quantity.

To find the equilibrium price, $p^\star$, we set $D(p^\star) = S(p^\star)$:

$$24-4p^\star = 4+6p^\star \iff 24-4 = 4p^\star+6p^\star \iff 20 = 10p^\star \iff p^\star = \frac{20}{10} \iff p^\star = 2$$

The quantity can be found by using $p^\star$ in either $S(p)$ or $D(p)$. Using demand:

$$q^\star = 24 - 4 \times 2 = 16$$

Using supply:

$$q^\star = 4 + 6 \times 2 = 16$$

Now the government imposes a per-unit tax (a quantity tax) of 1 on the good.

(ii) **[6 Points]** Find the price the buyers pay, the price sellers receive and the new equilibrium quantity.

Now $P_D = P_S + 1$. We can replace p in the demand function with $P_S + 1$ and then set demand equal to supply:

$$24 - 4(P_S + 1) = 4 + 6P_S$$
$$24 - 4P_S - 4 = 4 + 6P_S$$
$$16 = 10P_S$$
$$P_S = 1.6$$

Using $P_D = P_S + 1$ we can find the price buyers pay:

$$P_D = 1.6 + 1 = 2.6$$

The price increased for buyers by 60¢ and the price decreased for sellers by 40¢.

The new equilibrium quantity can be found using either the inverse demand or supply function. Using demand:

$$q^\star = 24 - 4P_D = 2t - 4 \times 2.6 = 24 - 10.4 = 13.6$$

Using supply:

$$q^\star = 4 + 6P_S = 4 + 6 \times 1.6 = 4 + 9.6 = 13.6$$

The tax decreased the equilibrium quantity by 2.4 units.

(iii) **[4 Points]** What portion of the tax do consumers pay and what portion of the tax do sellers pay?

Since the price increased by $\frac{3}{5}$ of the tax for consumers, they pay 60% of the tax. The price decreased by $\frac{2}{5}$ of the tax for sellers so they pay 40% of the tax.