Question 1 – (12 Points)

The following parts (i), (ii) and (iii) of this question are *unrelated*. Consider each separately.

(i) **(4 Points)** John says he strictly prefers $(x_1, x_2) = (2,3)$ to $(x_1, x_2) = (1,3)$. John also says he strictly prefers $(x_1, x_2) = (1,3)$ to $(x_1, x_2) = (3,1)$. Using this, come up with a preference relation between two bundles such that John would violate transitivity. Explain why.

Transitivity says that John should prefer $(x_1, x_2) = (2,3)$ to $(x_1, x_2) = (3,1)$. So a scenario that would violate transitivity is if $(x_1, x_2) = (3,1)$ was strictly prefered to $(x_1, x_2) = (2,3)$.

(ii) **(4 Points)** Mary says she strictly prefers $(x_1, x_2) = (1,2)$ to $(x_1, x_2) = (100,1)$. Do Mary's preferences violate monotonicity? Why/why not?

It does not violate montonicity. Monotonicity says that $\left(x_1', x_2'\right) \succ (x_1, x_2)$ if either

- $x_1' > x_1$ and $x_2' \geq x_2$, or
- $x_1' \geq x_1$ and $x_2' > x_2$.

Here, neither bundle has at least as much of one good and strictly more of at least one, so it doesn't violate monotonicity.

(iii) **(4 Points)** Sarah says she is indifferent between the bundles $(x_1, x_2) = (1,9)$ and $(x_1, x_2) = (9,1)$. Using this, come up with a preference relation between two bundles such that Sarah would violate convexity. Explain why.

Convexity says that the average of the two indifferent bundles should be at least as good, i.e. $(5,5) \succeq (1,9)$ and $(5,5) \succeq (9,1)$. An example that would violate convexity would then be if $(x_1, x_2) = (1,9)$ was *strictly* preferred to $(x_1, x_2) = (5,5)$.

Question 2 – (15 Points)

Your utility function for goods 1 and 2 is:

$$u(x_1, x_2) = x_1^{\frac{1}{2}} + \frac{1}{2}x_2$$

- **(7 Points)** If the prices of goods 1 and 2 are $p_1 = 1$ and $p_2 = 2$ and you have \$10 to spend, how much will you buy of each good?

  The *MRS* is:

  $$MRS = -\frac{\frac{1}{2}x_1^{-\frac{1}{2}}}{\frac{1}{2}} = -\frac{1}{x_1^{\frac{1}{2}}}$$

  Setting $|MRS| = \frac{p_1}{p_2}$ gives us the demand function for good 1:

  $$\frac{1}{x_1^{\frac{1}{2}}} = \frac{p_1}{p_2} \qquad \Longleftrightarrow \qquad x_1(p_1, p_2, m) = \left(\frac{p_2}{p_1}\right)^2$$

  With the prices, the demand for good 1 is then $x_1 = \left(\frac{2}{1}\right)^2 = 4$. If you buy 4 of good 1 at \$1 each, you have \$6 left to spend on good 2. Therefore you will buy 3 units of good 2.

- **(3 Points)** Is good 1 normal, inferior, or neither?

  The demand for good 1 does not depend on income so it is neither normal nor inferior.

- **(5 Points)** What is the own-price elasticity of demand for good 1? For this you will need to find the demand function $x_1(p_1, p_2, m)$. Is it inelastic, elastic or unit elastic?

$$\varepsilon_1 = \frac{dx_1(p_1, p_2, m)}{dp_1}\frac{p_1}{m} = -2 \times \frac{p_2^2}{p_1^3} \times \frac{p_1^2}{\frac{p_2^2}{p_1^2}} = -2$$

  Good 1 is elastic.

Question 3 – (9 Points)

A firm cuts down enormous trees and sells the wood. To cut down a tree it needs 2 sawyers (workers who saw down trees for a living) and a giant two-person saw. One sawyer is not able to use a saw on their own, as the saws are very big. If there are 3 sawyers with only 1 saw, the third sawyer will be idle (they can't help as the saw can only be used be two people).

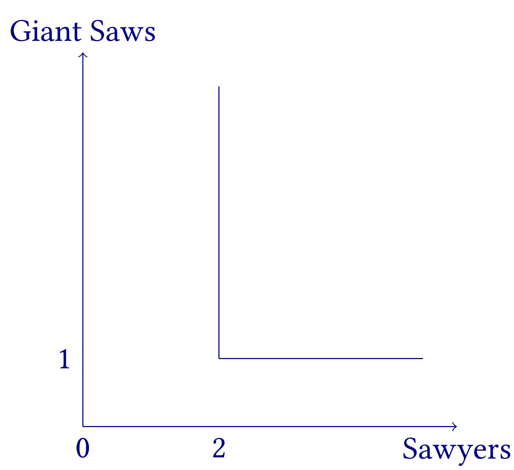
(i) **(5 Points)** Call the sawyers x_1 and the giant saws x_2. Given the information provided, what is the production function $f(x_1, x_2)$ for cutting down trees?

The sawyers and giant saws are perfect complements and need to be used in a 2:1 ratio. The production function is then:

$$f(x_1, x_2) = \min\left\{\frac{1}{2}x_1, x_2\right\}$$

(ii) **(4 Points)** Sketch an isoquant for this production function.

An isoquant for one unit of output would be as follows:



Question 4 – (18 Points)

The cost function for a firm in a perfectly competitive industry is given by:

$$c(q) = q^2 + 1$$

All firms have the same cost function. The market demand function for the good sold in this industry is:

$$D(p) = 100 - 10p$$

(i) **(2 Points)** What is each firm's average cost function?

$$AC(q) = \frac{c(q)}{q} = \frac{q^2+1}{q} = q + \frac{1}{q}$$

(ii) **(2 Points)** What is each firm's marginal cost function?

$$MC(q) = \frac{dc(q)}{dq} = 2q$$

(iii) **(7 Points)** There are 30 firms in the industry in the short run. What is the industry supply curve? What is the equilibrium market price? How much will each individual firm produce? What will each firm's profits/losses be?

Since in perfect competition, $p = MC$, each individual firm's supply function, $S_i(p)$, can be found as follows:

$$p = MC(q) \quad \Longleftrightarrow \quad p = 2q \quad \Longleftrightarrow \quad S_i(p) = \frac{p}{2}$$

The industry supply function is then $S(p) = 30 \times \frac{p}{2} = 15p$. To find the market price in equilibrium, we set $S(p) = D(p)$:

$$15p = 100 - 10p \quad \Longleftrightarrow \quad p = 4$$

With $p = 4$, each firm will want to produce $S_i(p) = \frac{4}{2} = 2$ units. Each firm's profits will then be:

$$\pi = p \times q - q^2 - 1 = 4 \times 2 - 2^2 - 1 = 3$$

Each firm makes profits of 3.

(iv) **(7 Points)** How many firms will be in this industry in the long run?

We know in equilibrium that each firm should be earning zero profits. Call the price that makes profits zero $p^\star$. Then

$$\begin{aligned} \pi &= 0 \\ p^\star \times \frac{p^\star}{2} - \left(\frac{p^\star}{2}\right)^2 - 1 &= 0 \\ \frac{(p^\star)^2}{4} - 1 &= 0 \\ p^\star &= 2 \end{aligned}$$

So the price in the long run must be 2. At a price of 2, each firm will produce $S_i(p^\star) = \frac{2}{2} = 1$ unit of output. At a price of 2, the total quantity bought and sold will be $D(p^\star) = 100 - 10 \times 2 = 80$. Therefore there will be 80 firms in the long run.

Question 5 – (15 Points)

You are considering developing a new piece of software currently unavailable in the world. If you develop it, you will patent it and then you will have monopoly rights to sell that software. You have \$200 sitting in your bank account and the development of the software will cost \$100. Your friend, who is a marketing expert, tells you that the demand for your software will be:

$$D(p) = 50 - \frac{1}{2}p,$$

where p is the price per download of the software. Each additional unit of software sold will incur you no cost.

However, you are concerned that the government will regulate the sale of your software. Your friend in the government gives you the following information.

- There is a $\frac{1}{3}$ probability that the government will not regulate you at all, and you will be free to choose any price you wish.
- There is a $\frac{1}{3}$ probability that the government will force you to charge at marginal cost.
- There is a $\frac{1}{3}$ probability that the government will force you to charge a price such that you will break even.

If your utility for wealth is $u(W) = \sqrt{W}$, will you decide to go ahead and develop the software?

(i) **(10 Points)** What profit will you make from each of the three possible government actions (no regulation and two types of regulation)?

No Regulation Case.
The inverse demand curve is:

$$p(q) = 100 - 2q$$

The marginal revenue function is then:

$$MR(q) = 100 - 4q$$

Since marginal cost is zero, the optimal quantity is where $MR(q) = 0$, so $q_1 = 25$. At $q_1 = 25$, the price will be $p_1 = 100 - 2 \times 25 = 50$. Therfore the profit will be:

$$\pi_1 = 50 \times 25 - 100 = 1150$$

Regulated To Price At Marginal Cost.
If $p = MC$, then $p_2 = 0$. The revenue will be zero. The monopolist will make a loss equal to the fixed cost, so $\pi_2 = -100$.

Regulated To Price Such That Profits Are Zero.
In this case, profits are zero by construction, so $\pi_3 = 0$.

(ii) **(5 Points)** If your utility for wealth is $u(W) = \sqrt{W}$, will you decide to go ahead and develop the software? Show why or why not.

Your expected utility from developing the software is then:

$$\frac{1}{3}u(\pi_1 + 200) + \frac{1}{3}u(\pi_2 + 200) + \frac{1}{3}u(\pi_3 + 200) = \frac{1}{3}\sqrt{1350} + \frac{1}{3}\sqrt{100} + \frac{1}{3}\sqrt{200} = 20.29483$$

If you do nothing and just keep the \$200, you will get expected utility $\sqrt{200} = 14.14214$. Therefore you should develop the software.

Question 6 – (15 Points)

An art gallery is frequented by both art students and rich art lovers. The inverse demand curve for both groups for entry into the art gallery is:

$$p_S(q_S) = 20 - q_S$$
$$p_R(q_R) = 40 - q_R$$

The art gallery has the following cost function:

$$c(q_S, q_R) = 10(q_S + q_R) + 10$$

(i) **(7 Points)** Suppose first that the art gallery can't tell the two groups apart. What price should it charge for entry? How many people will visit the art gallery? What will the art gallery's profits be? What is the consumer surplus?

The two demand functions are:

$$D_S(p) = 20 - p$$
$$D_R(p) = 40 - p$$

Therefore the market demand is:

$$D(p) = D_S(p) + D_R(p) = 20 - p + 40 - p = 60 - 2p$$

The inverse market demand is then:

$$p(q) = 30 - \frac{1}{2}q$$

The marginal revenue is then $MR(q) = 30 - q$. Marginal cost is $MC(q) = 10$. Setting these equal and solving for q gives $q = 20$. The price will then be $p = 30 - \frac{1}{2} \times 20 = 20$. Profits are then:

$$\pi = (20 - 10) \times 20 - 10 = 190$$

Consumer surplus is the area under the inverse demand curve, above marginal cost and up to the quantity sold. This is the triangle with area:

$$CS = \frac{1}{2} \times (30 - 20) \times 20 = 100$$

(ii) **(6 Points)** Now suppose the art gallery can charge different prices to the different groups. The students will show that they are students by showing their student cards. What prices should the art gallery charge to both groups? How many of each group will visit the art gallery? What will the art gallery's profits be? What will the total consumer surplus be?

Now we just maximize profits from each group separately. $MR_S(q_S) = 20 - 2q_S$ and $MC_S(q_S) = 10$. Setting these equal gives $q_S = 5$. The price is then $p_S = 20 - 5 = 15$.

For the other group, $MR_R(q_R) = 40 - 2q_R$ and $MC_R(q_R) = 10$. Setting these equal gives $q_R = 15$. The price is then $p_S = 40 - 15 = 25$.

Total profits are then:

$$\pi = p_S q_S + p_R q_R - 10 \times (q_S + q_R) - 10 = 15 \times 5 + 25 \times 15 - 10 \times (5 + 15) - 10 = 240$$

Consumer surplus is the sum of consumer surplus from the two groups:

$$CS = \frac{1}{2} \times (20 - 15) \times 5 + \frac{1}{2} \times (40 - 25) \times 15 = 12.5 + 112.5 = 125$$

The consumer surplus is actually larger under price discrimination.

(iii) **(2 Points)** Suppose the art gallery can't tell the groups apart and now decided to offer two types of gallery experiences. One gives only entry but the other also includes a tour. The prices and quality of two different entry tickets are designed such that the students self select into the ticket that doesn't include a tour and the rich group self selects into the more ticket that does include the tour. What degree of price discrimination would this be? (No calculation is required).

This would be second-degree price discrimination.

Question 7 – (18 Points)

Answer the following questions about these games.

(i) **(6 Points)** Find all the Nash equilibria (pure and mixed) of the following game:

	ℓ	r
u	2, 2	0, 5
d	5, 0	1, 1

The unique Nash equilibrium is (d, r). Suppose we tried to find a mixed strategy equilibrium. Player 2 players ℓ with probability q. Then we need:

$$2q + 0(1 - q) = 5q + (1 - q) \iff q = -\frac{1}{2}$$

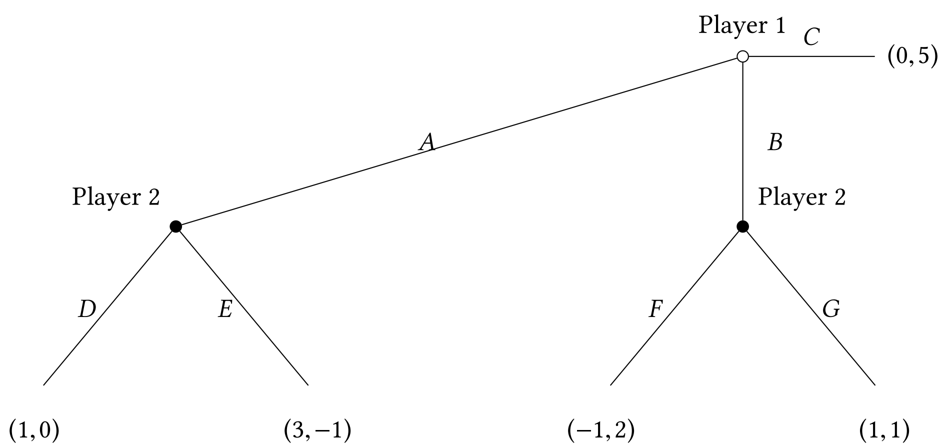
which is impossible. Therefore there is no mixed strategy Nash equilibrium of this game.

(ii) **(6 Points)** Find all the Nash equilibria (pure and mixed) of the following game:

	L	R
U	$3, -3$	$-1, 2$
D	$-3, 4$	$5, -6$

There are no pure strategy equilibria in this game. The unique Nash equilibrium is a mixed strategy equilibrium where Player 1 plays U with probability $p = \frac{2}{3}$ and Player 2 plays L with probability $q = \frac{1}{2}$.

(iii) **(6 Points)** Find the subgame-perfect Nash equilibrium of this extensive-form game:



Player 2 will choose D over E and F over G. Player 1 then will choose A because with A it gets 1, while with B it would get -1 and with C it would get 0.

Question 8 – (18 Points)

Two firms compete in a particular industry by simultaneously setting quantities. The market price is then determined from the sum of the two firms' quantities. They both have the same cost function:

$$c_1(q_1) = 4q_1 \qquad \text{and} \qquad c_2(q_2) = 4q_2$$

The inverse demand curve for the good that they sell is:

$$p(q) = 10 - q$$

where $q = q_1 + q_2$, the sum of the two firms' quantities.

(i) **(6 Points)** Find the Cournot equilibrium quantities (show your work, do not just write down the final answer). What profits will each firm make?

Firm 1's profit function is:

$$\pi_1(q_1, q_2) = (10 - q_1 - q_2) q_1 - 4q_1 = 6q_1 - q_1^2 - q_1 q_2$$

Firm 2's profit function is:

$$\pi_2(q_1, q_2) = (10 - q_1 - q_2)\, q_2 - 4q_2 = 6q_2 - q_2^2 - q_1 q_2$$

Maximizing with respect to the firm's own quantity:

$$\frac{d\pi_1(q_1, q_2)}{dq_1} = 6 - 2q_1 - q_2 = 0$$

$$\frac{d\pi_2(q_1, q_2)}{dq_2} = 6 - 2q_2 - q_1 = 0$$

Firm 1's reaction function is:

$$q_1(q_2) = 3 - \frac{q_2}{2}$$

Firm 2's reaction function is:

$$q_2(q_1) = 3 - \frac{q_1}{2}$$

The Cournot equilibrium is where both reaction functions intersect. Inserting $q_2(q_1)$ into $q_1(q_2)$ gives:

$$\begin{aligned} q_1 &= 3 - \frac{3 - \frac{q_1}{2}}{2} \\ q_1 &= \frac{3}{2} - \frac{q_1}{4} \\ \frac{3}{4} q_1 &= \frac{3}{2} \\ q_1 &= 2 \end{aligned}$$

Using $q_1 = 2$ in $q_2(q_1)$ gives $q_2 = 3 - \frac{2}{2} = 2$ also. The price will then be $p = 10 - 2 - 2 = 6$. Profits will then be:

$$\pi = 6 \times 2 - 4 \times 2 = 4$$

(ii) **(4 Points)** If both firms decided to collude and form a cartel in order to increase their profits, what quantity will they jointly produce and what profits would they get? (Assume both firms will produce the same output and share the cartel profits equally).

The two firms should choose q, the sum of their outputs. Once they have optimally chosen q each will produce $\frac{q}{2}$. The profit as a function of q is:

$$\pi(q) = (10 - q)\, q - 4q = 6q - q^2$$

Maximizing profits:

$$\frac{d\pi(q)}{dq} = 6 - 2q = 0 \qquad \Longrightarrow \qquad q = 3$$

So each firm should produce $q_1 = q_2 = 1.5$. The price will then be $p = 10 - 3 = 7$. Each firm will make profits:

$$\pi = 7 \times 1.5 - 4 \times 1.5 = 4.5$$

The firms now make profits 4.5 instead of 4.

(iii) **(4 Points)** If firm 1 were to cheat on the cartel, what quantity should it produce in order to maximize its profits (assuming firm 2 will stick to the cartel quantity)? What profit would it make by cheating?

Firm 1 will choose the quantity on its reaction curve, as this gives the maximum profits given any quantity of firm 2:

$$q_1 = 3 - \frac{1.5}{2} = 2.25$$

Firm 1 would like to cheat and produce a little more quantity. The price would then fall to $p = 10 - 1.5 - 2.25 = 6.25$. Firm 1's profits will then be:

$$\pi = 6.25 \times 2.25 - 4 \times 2.25 = 5.0625$$

(iv) **(4 Points)** Suppose that when they formed the cartel they said that if any firm cheated on the cartel they would both produce at the Cournot equilibrium forever. If the interest rate is 10%, would any firm firm want to cheat on the cartel? Show why or why not.

Sticking with the cartel earns 4.5 today and 4.5 forever afterwards. The present value of this is:

$$4.5 + \frac{4.5}{0.1} = 49.5$$

Cheating on the cartel earns 5.0625 today and then 4 forever afterwards. The present value of this is:

$$5.0625 + \frac{4}{0.1} = 45.0625$$

Therefore the firms should stick to the cartel.