Question 1 – Demand

Your demand functions for goods 1 and 2 are:

$$x_1(p_1, p_2, m) = \begin{cases} \frac{2m-100}{p_1} & \text{if } m \geq 50 \\ 0 & \text{if } m < 50 \end{cases}$$

$$x_2(p_1, p_2, m) = \begin{cases} \frac{100-m}{p_2} & \text{if } m \leq 100 \\ 0 & \text{if } m > 100 \end{cases}$$

Assume that your income is always such that $50 \leq m \leq 100$ so we can write demand as:

$$x_1(p_1, p_2, m) = \frac{2m - 100}{p_1}$$

$$x_2(p_1, p_2, m) = \frac{100 - m}{p_2}$$

(i) Is good 1 an ordinary or a Giffen good?

(ii) Is good 2 an ordinary or a Giffen good?

(iii) Is good 1 a normal good or an inferior good?

(iv) Is good 2 a normal good or an inferior good?

(v) Is good 1 a substitute, a complement, or neither, for good 2?

Question 2 – Production

A firm cuts down enormous trees and sells the wood. To cut down a tree it needs 2 sawyers (workers who saw down trees for a living) and a giant two-person saw. One sawyer is not able to use a saw on their own, as the saws are very big. If there are 3 sawyers with only 1 saw, the third sawyer will be idle (they can't help as the saw can only be used be two people).

(i) Call the sawyers x_1 and the giant saws x_2. Given the information provided, what is the production function $f(x_1, x_2)$ for cutting down trees?

(ii) Sketch an isoquant for this production function.

Question 3 – Uncertainty and Monopoly

You are considering developing a new piece of software currently unavailable in the world. If you develop it, you will patent it and then you will have monopoly rights to sell that software. You have \$200 sitting in your bank account and the development of the software will cost \$100. Your friend, who is a marketing expert, tells you that the demand for your software will be:

$$D(p) = 50 - \frac{1}{2}p,$$

where p is the price per download of the software. Each additional unit of software sold will incur you no cost.

However, you are concerned that the government will regulate the sale of your software. Your friend in the government gives you the following information.

- There is a $\frac{1}{3}$ probability that the government will not regulate you at all, and you will be free to choose any price you wish.
- There is a $\frac{1}{3}$ probability that the government will force you to charge at marginal cost.
- There is a $\frac{1}{3}$ probability that the government will force you to charge a price such that you will break even.

Answer the following questions:

(i) What profit will you make from each of the three possible government actions (no regulation and two types of regulation)?

(ii) If your utility for wealth is $u(W) = \sqrt{W}$, will you decide to go ahead and develop the software? Show why or why not.

Question 4 – Price Discrimination

An art gallery is frequented by both art students and rich art lovers. The inverse demand curve for both groups for entry into the art gallery is:

$$p_S(q_S) = 20 - q_S$$
$$p_R(q_R) = 40 - q_R$$

The art gallery has the following cost function:

$$c(q_S, q_R) = 10(q_S + q_R) + 10$$

(i) Suppose first that the art gallery can't tell the two groups apart. What price should it charge for entry? How many people will visit the art gallery? What will the art gallery's profits be? What is the consumer surplus?

(ii) Now suppose the art gallery can charge different prices to the different groups. The students will show that they are students by showing their student cards. What prices should the art gallery charge to both groups? How many of each group will visit the art gallery? What will the art gallery's profits be? What will the total consumer surplus be?

(iii) Suppose the art gallery can't tell the groups apart and now decided to offer two types of gallery experiences. One gives only entry but the other also includes a tour. The prices and quality of two different entry tickets are designed such that the students self select into the ticket that doesn't include a tour and the rich group self selects into the more ticket that does include the tour. What degree of price discrimination would this be? (No calculation is required).